lule 2

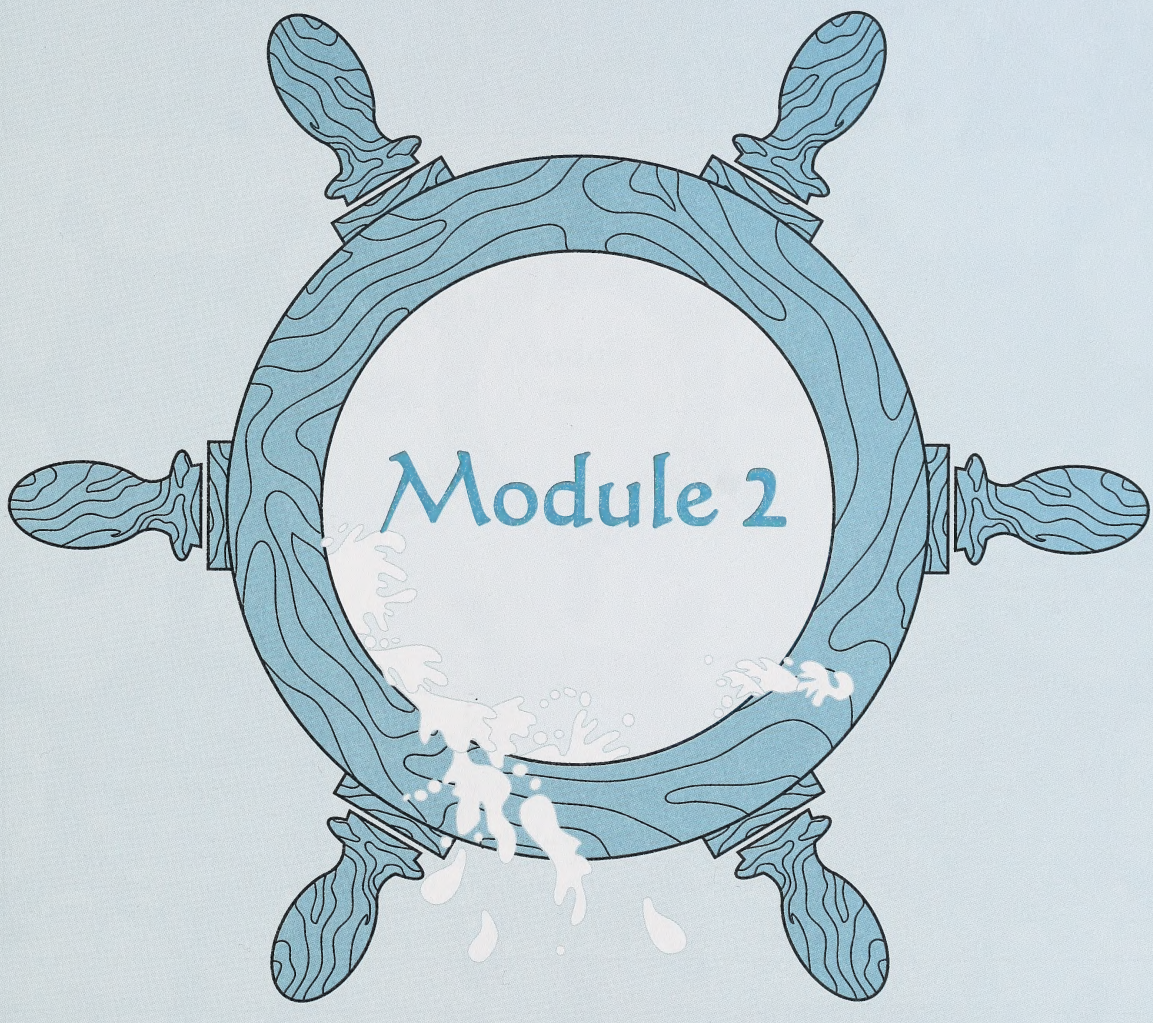
Mathematics 5

Patterns

Learning Technologies Branch

Alberta LEARNING

Mathematics 5



Patterns

Learning Technologies Branch

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Mathematics 5
Module 2: Patterns
Student Module Booklet
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome Aboard
Mathematics 5

Ahoy! Mathematics 5 contains nine modules.
You should work through the modules in order (from 1 to 9) because concepts and skills introduced in one module will be reinforced, extended, and applied in later modules.

Adventures on the High Seas

Kassidy: Connor, guess what? I noticed an advertisement in the newspaper. A second-hand bookshop just opened, and it's supposed to have some really old books from all over the world.

Connor: Some may even be older than our great-grandparents! Let's go take a look.

Excited about what they might find to read, Connor and Kassidy hurry off to the bookshop.

Connor: Wow! Look at this, Kassidy. Here's an actual journal kept by Captain Quinn almost 300 years ago. It's kind of musty, and parts are hard to read, but look at how much math the captain used!

Kassidy: There are also maps, descriptions of ships, and reports about sea conditions.

Connor: Let's buy it. Figuring out what the journal says will be like unravelling a mystery. Maybe it will have clues about real buried treasure.

From time to time throughout the course, you will work with many of the facts and figures about the days of sailing boats and sea captains that Kassidy and Connor discover.

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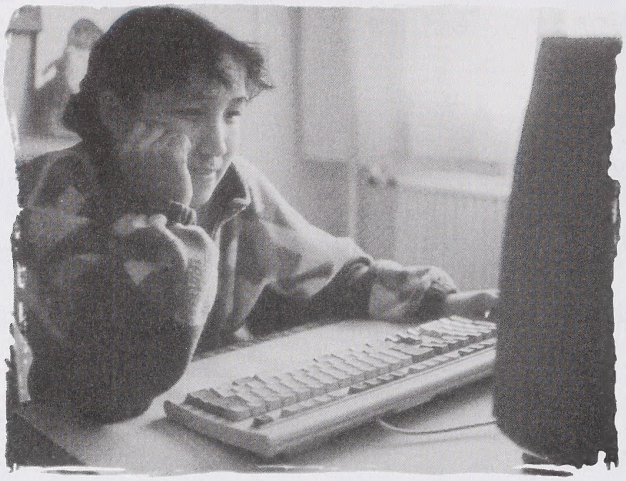
Course Features



BRYAN AND CHERRY ALEXANDER PHOTOGRAPHY

Take the time to look through the Student Module Booklet and the Assignment Booklets and notice the following design features.

- Each module has a Module Overview, Module Summary, and Review.
- Each module has several lessons. Each lesson focuses on a big idea that is central to the topic being learned in the module.
- Each lesson has several activities. An activity in each lesson is related to the high seas adventure theme. The last activity in each lesson is a Challenge Activity.
- Each module has a Glossary and an Answer Key in the Appendix. In several modules there are also special pull-out pages in the Appendix.
- Each module has special exercises that focus on certain mathematical skills. For example, The Numbers in the News project involves a scavenger hunt for samples of math in everyday life. The Keystrokes exercise introduces some "funky features" of the calculator that can be used to explore and practise important number ideas.

Required Resources

There are no spaces provided in the Student Module Booklets for your answers. This means you will need a binder and loose-leaf paper or a notebook to do your work.

In order to complete the course, you will need a copy of the Mathematics 5 textbook, *Quest 2000: Exploring Mathematics*, the soft-cover book *Quest 2000: Exploring Mathematics: Practice and Homework Book*, a basic four-operation calculator (such as the TI-108 calculator), and various manipulatives (base ten blocks and pattern blocks).

If you wish to complete the optional computer activities, you must have access to a computer that is connected to the Internet.

Visual Cues

For your convenience, the most important mathematical rules and definitions are highlighted. Icons are also used as visual cues. Each icon tells you to do something.

Use your calculator.

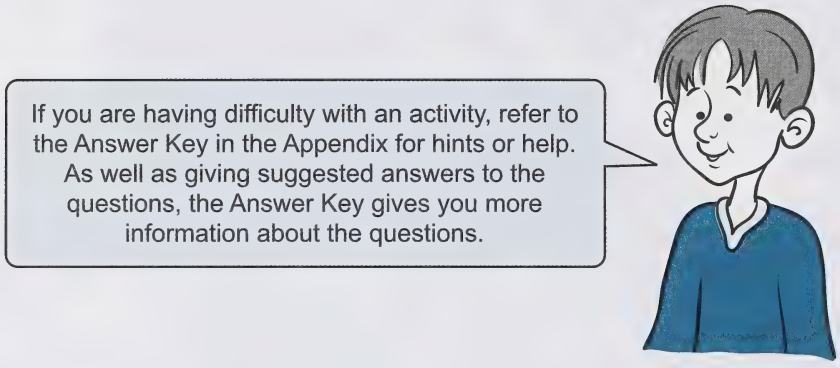
Use the Internet.

Refer to the textbook or the Practice and Homework Book.

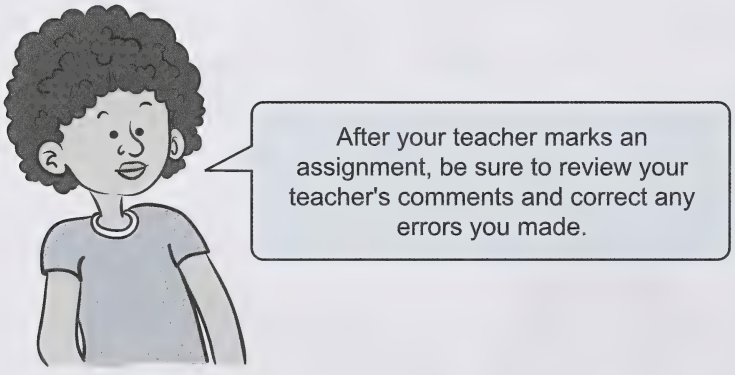
Your guides for this course are Kassidy and Connor.

Assessment and Feedback

The Mathematics 5 course is carefully designed to give you many opportunities to discover how well you are doing. In every activity you will be asked to turn to the Appendix to check your answers. Completing the activities and comparing your answers to the suggested answers in the Appendix will help you better understand math concepts, develop math skills, and improve your ability to communicate mathematically and solve problems.



Twice in each module you will be asked to give your teacher your completed assigments to mark. Your teacher will give you feedback on how you are doing.



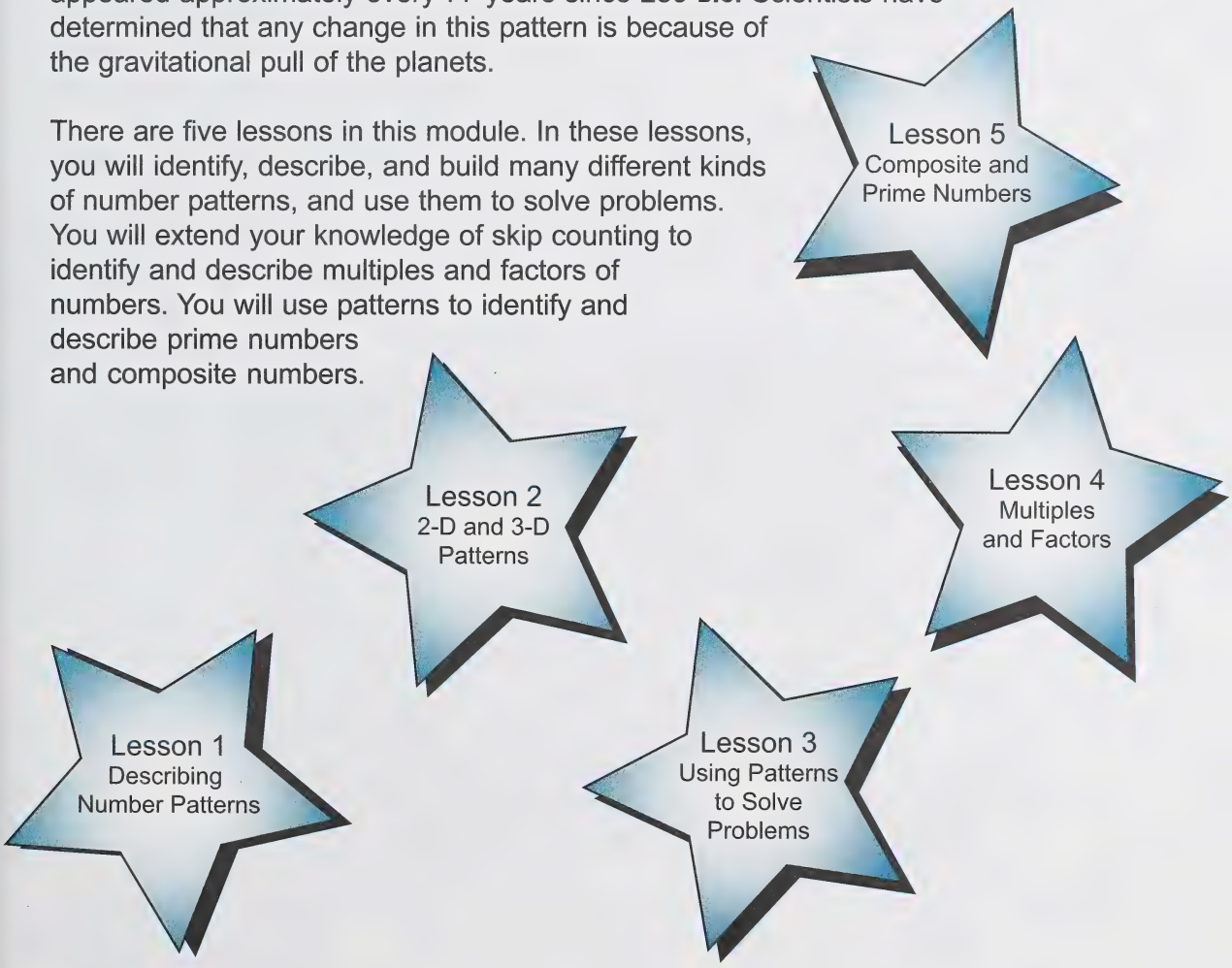
There will be a final test at the end of the course. You can prepare for the final test by completing the Review at the end of each module.

Module Overview

Captain Quinn and his crew enjoyed pointing out the different patterns of stars in the night sky. Their favourite star groupings were the Big Dipper and the Little Dipper. They noticed that the two dippers seemed to revolve around a star, the North Star, which didn't appear to move. The sailors used the North Star, located at the tip of the handle of the Little Dipper, as a guide in their travels.

Another person who noticed patterns in the sky was the astronomer Edmund Halley. In 1705, Halley concluded that a major comet had appeared in Earth's sky in 1531, 1607, and 1682. He predicted the comet would reappear in about 1758. The comet, which became known as Halley's comet, returned in 1759. Records show that this comet has appeared approximately every 77 years since 239 B.C. Scientists have determined that any change in this pattern is because of the gravitational pull of the planets.

There are five lessons in this module. In these lessons, you will identify, describe, and build many different kinds of number patterns, and use them to solve problems. You will extend your knowledge of skip counting to identify and describe multiples and factors of numbers. You will use patterns to identify and describe prime numbers and composite numbers.



Your mark on this module will be determined by how well you complete the two Assignment Booklets.

The mark distribution is as follows:

Assignment Booklet 2A

Lesson 1 Assignment	14 marks
Lesson 2 Assignment	21 marks
Lesson 3 Assignment	22 marks

Assignment Booklet 2B

Lesson 4 Assignment	16 marks
Lesson 5 Assignment	17 marks
Numbers in the News	10 marks
Total	100 marks

When doing the assignments, work slowly and carefully. Be sure you attempt each part of the assignments. If you are having difficulty, you may use your course materials to help you, but you must do the assignments by yourself.

You will submit Assignment Booklet 2A to your teacher before you begin Lesson 4. You will submit Assignment Booklet 2B to your teacher at the end of this module.

Numbers in the News

Numbers are everywhere! Newpapers and magazines are full of stories and advertisements that show how numbers are used every day.

The following Scavenger Hunt asks you to look through newspapers and magazines for samples of number ideas like those you will be using in this module. Read through the list now and begin by collecting samples of the number ideas you already understand. You may collect other samples as you learn about them in the module.

Scavenger Hunt

Cut out articles or advertisements from newspapers or magazines that show patterns or relationships among numbers being used in different situations. For each of your examples, explain the pattern you see. Here are some suggestions of things to look for:

1. a pattern that repeats two or more geometric shapes
2. a table (chart) that shows how numbers grow or are related in some way
3. graphs that show patterns or trends from which predictions can be made
4. situations that show the use of multiples, factors, primes, or composites
5. objects that are sorted in different ways
6. problems or puzzles that use logical reasoning

You will find further instructions for submitting your project in Assignment Booklet 2B.

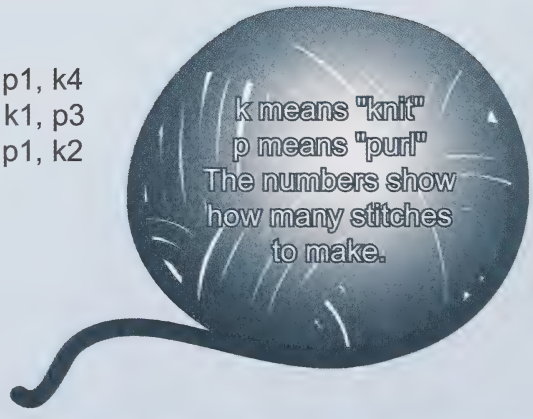
Lesson 1

Describing Number Patterns

BRYAN AND CHERRY ALEXANDER PHOTOGRAPHY

Have you ever watched someone knitting a sweater? Have you ever knitted a scarf? You can make very complicated designs by using only two kinds of stitches. For example, following is the sequence of stitches you can use to make a diamond pattern:

Row 1: k6, p1, k6
Row 2: p5, k1, p1, k1, p5
Row 3: k4, (p1, k1) 2 times, p1, k4
Row 4: p3, (k1, p1) 3 times, k1, p3
Row 5: k2, (p1, k1) 4 times, p1, k2
Row 6: (p1, k1) 6 times, p1
Row 7: Repeat row 6.
Row 8: Repeat row 6.
Row 9: Repeat row 5.
Row 10: Repeat row 4.
Row 11: Repeat row 3.
Row 12: Repeat row 2.



There are many other patterns in everyday life. Patterns can be found in actions, shapes, and numbers.

In this lesson you will describe and build patterns. You will discover that tables are useful tools for identifying, recording, and extending patterns.

Activity 1

Today you will explore and describe patterns.

How much we enjoyed our fifth voyage!
We happened along a beautiful but strange island.
Let me tell you more. . . .

Meeker, my ship's cook, went exploring while the rest of the crew relaxed on the warm, sandy beach that wrapped all around the island's perimeter.

- Captain Quinn

Meeker came to a stream that had stepping stones spaced evenly in a row across it. Instead of stepping from one stone to the next, Meeker hopped across the stream. Turn to page 10 of your textbook and examine the number line that shows Meeker's hopping pattern.

The tick marks at the ends of the number line stand for the opposite shores of the stream. The tick marks in between stand for the stepping stones in the stream. The arcs show how Meeker hopped across the stream. The dots indicate the stones Meeker stepped on.

1. **a.** Copy the number line (shown on page 10 of your textbook) into your notebook. Number the tick marks from left to right, starting with the number 0. Number the arcs from left to right, starting with 1.

   b. Describe Meeker's hopping pattern.

   c. How long was Meeker's last hop to shore?

   d. How many hops would it have taken Meeker to cross the stream? Explain.

2. Imagine that Meeker's hopping pattern had been 1 stone, 2 stones, 3 stones, and so on, so that each hop was one stone longer than the hop before.

   a. Make a number line to record the hopping pattern and show the stones that Meeker would have landed on. Number the tick marks and the arcs as you did in question 1. List the stones Meeker used in the first six hops.

   b. How long was Meeker's last hop to shore? Explain.

   c. How many hops would it have taken for Meeker to cross the stream? Explain.

   d. Describe a pattern that tells how the length of any particular hop is related to the hop number (first hop, second hop, third hop, and so on).

e. Still using the same pattern, what is the greatest number of stepping stones that could have been in the stream if Meeker needed to make exactly 10 hops to cross it? Explain.

Check your answers on pages 84 and 85 in the Appendix.

In the Chinese culture, years are named after different animals. These years occur in a regular pattern. For example, 1932, 1944, 1956, 1968, 1980, and 1992 were Years of the Monkey.

You may use a calculator to find the number pattern for the Years of the Monkey.

1932, 1944, 1956, 1968, 1980, 1992

+12 +12 +12 +12 +12

Extending this pattern, the next three Years of the Monkey are 2004, 2016, and 2028.

3. Turn to page 38 of the textbook and answer question 1 of Skill Bank from This Unit. You may use a calculator.

Check your answers on page 85 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Consider the number pattern 2, 3, 5, 8, ...

- Describe how the pattern grows if the fifth number in the pattern is 12.
- Describe how the pattern grows if the fifth number in the pattern is 13.

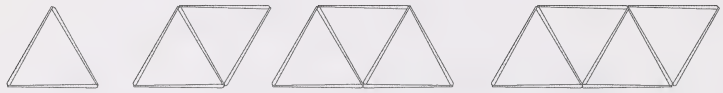
Discuss the answers with your home instructor.

Activity 2

Today you will build and extend more patterns.

You will need some toothpicks for this activity.

1. **a.** Use toothpicks to copy these triangular shapes, and then build the next three designs.



b. How many toothpicks are needed to build the shape with five triangles in a row?

c. Copy and complete a table like this one to record the number of triangles and the matching number of toothpicks in your constructions.

Number of Triangles	1	2	3							10
Number of Toothpicks	3									

d. Describe how the pattern grows.

e. If you know how many triangles there are, how can you determine the number of toothpicks?

f. Predict the number of toothpicks needed to make 75 triangles. Explain how you arrived at your answer.

Check your answers on page 86 in the Appendix.

Baby-Sitter Course

A baby-sitting course is given twice yearly for students in Grades 5 to 8. Safety and responsible child care are emphasized. For additional information, please telephone **555-1234.**

Do you ever baby-sit? What are baby-sitters in your community paid?

2. a. Turn to page 11 of your textbook. Copy the information given in question 3 of Starting Out into a table like this one. Don't complete the table at this time.

Hours Worked	1	2	3	4	5	6	7	8	9	10
Pay										

b. Describe the pattern that tells how Kevin's pay increases as he works more hours.

c. How would Kevin's pay change if he worked less hours?

d. If you know how many hours Kevin works, how can you find how much he will be paid? Explain.

e. Complete the table.

3. How do tables help you to understand patterns?

Check your answers on page 86 in the Appendix.

Sharing Time

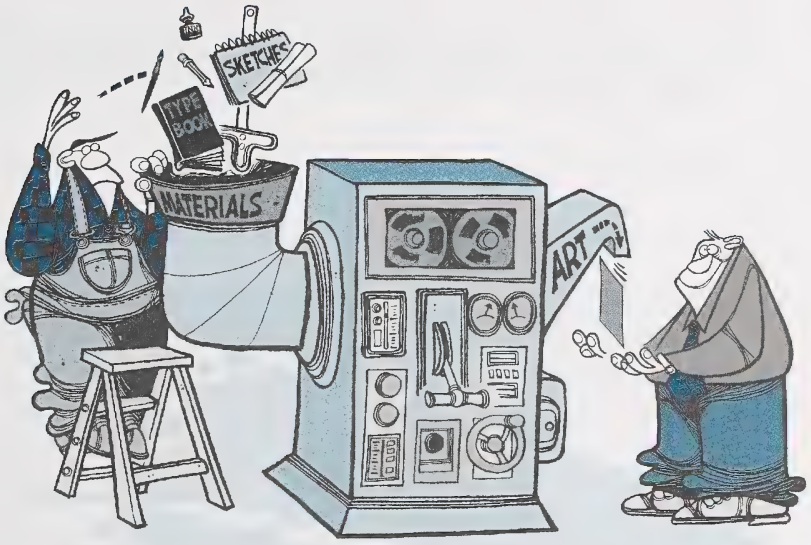
Now it's time to show your home instructor what you have been learning about patterns.

Turn to page 5 in the Practice and Homework Book and complete questions 1 to 3. You will need toothpicks.

Your home instructor will help you check your answers.

Activity 3

Today you will work with Input/Output rules.



This is an illustration of an imaginary Input/Output machine. Input/Output machines are similar to pasta makers, sausage machines, and cookie presses. They take the raw materials and work on them somehow to produce a new material.

Turn to page 23 of the textbook and look at the illustrations of the two Number Cruncher machines. You can use the guess-and-check strategy to find the Input/Output rule for each Number Cruncher machine.

For example, the first Number Cruncher machine uses the Input/Output rule **multiply the number by 2**.

$2 \times 4 = 8$

$2 \times 5 = 10$

$2 \times 6 = 12$

$2 \times 8 = 16$

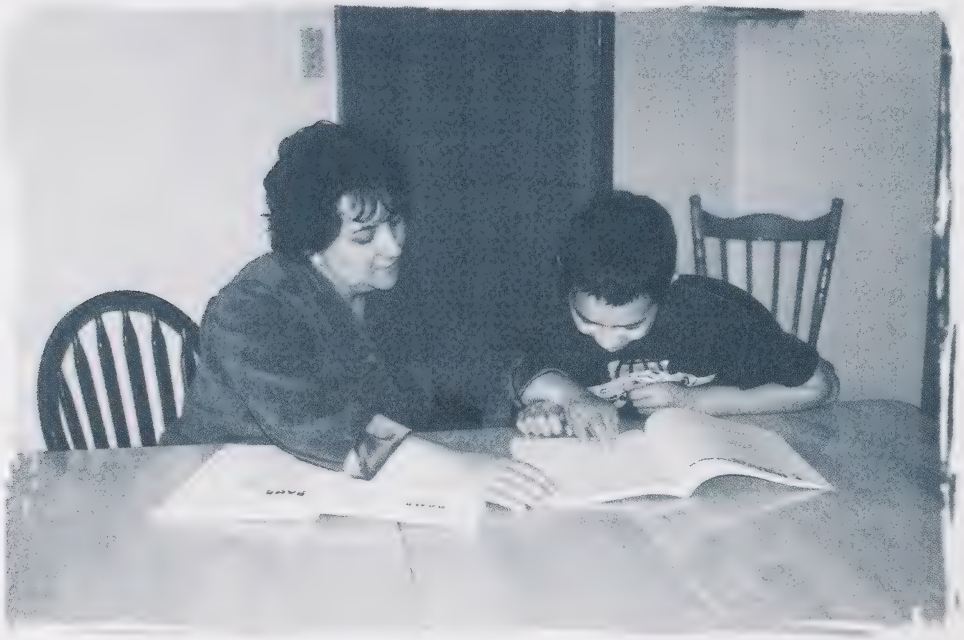
1. What is the Input/Output rule for the second Number Cruncher machine on page 23?

2. Turn to page 10 of the Practice and Homework Book and complete questions 1 to 6.

Check your answers on page 87 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

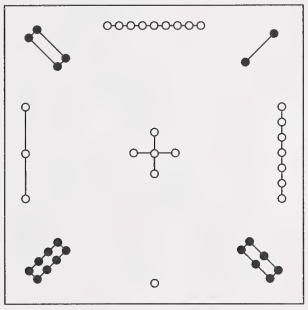


Turn to page 11 of the Practice and Homework Book and complete questions 1 to 4.

Your home instructor will help you check your answers.

Challenge Activity

There is a legend that the Chinese Emperor Yu, who lived about 4500 years ago, copied the Lo-Shu pattern from the design on the back of a sacred turtle found in the Lo River. Following are two versions of the Lo-Shu pattern and its equivalent number form.



4	9	2
3	5	7
8	1	6

In the Lo-Shu pattern, each set of markings represents a number. The sum of the numbers in each row, column, or diagonal is 15.

For more information on the Lo-Shu pattern, go to the following website:

http://forum.swarthmore.edu/alejandre/magic.square/loshu2.html

People in many different cultures are interested in patterns. These patterns may go around in circles, squares, and triangles.

Turn to pages 26 and 27 of your textbook.

1. Examine the Japanese Number Circle on the bottom of page 27 in your textbook and describe the pattern.

   Note: The top spoke of the Japanese Number Circle should read 47, **48**, 49, 50, 51.

Check your answer on page 87 in the Appendix.

Look at the Tibetan Number Square on page 26. The photograph isn't very clear, but there are three sections to the square:

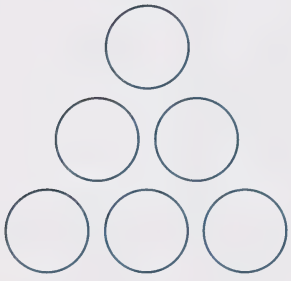
- The outer section is divided into 12 squares, each containing one of the 12 animals representing years: the Rat, the Ox, the Tiger, the Rabbit, the Dragon, the Snake, the Horse, the Ram, the Monkey, the Rooster, the Dog, and the Pig.

- The middle section is made of eight squares, each containing one of the symbols representing the eight elements: fire, earth, lake, heaven, water, mountain, thunder, and wood.

- The inner section contains nine squares arranged in a 3×3 array. The squares contain the numbers 1 through 9, with 5 in the centre. The numbers are arranged so that the sum of the numbers in each column, row, or diagonal is 15.

4	9	2
3	5	7
8	1	6

2. For each of the questions below, copy the following triangular pattern of circles. Then use all of the numbers 1, 2, 3, 4, 5, and 6 in each triangle pattern. Write one number in each circle to get the required sum.

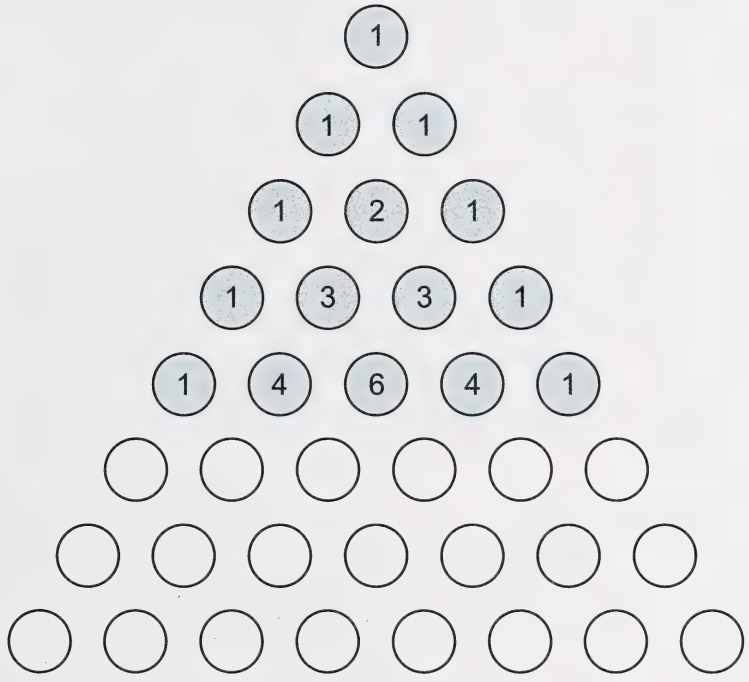


a. The sum of the numbers on each side is 9.
b. The sum of the numbers on each side is 10.
c. The sum of the numbers on each side is 11.
d. The sum of the numbers on each side is 12.

Check your answers on page 88 in the Appendix.

Look at the Chinese triangle on page 27 of the textbook. This Chinese triangle is very similar to another famous triangle pattern called Pascal's Triangle. However, the Chinese Triangle was developed 300 years before Pascal invented his triangle.

3. a. Copy and complete the following triangle. Explain how you arrived at your answer.



b. Find the sum of each horizontal row. Is there a pattern? Explain.

Check your answers on page 88 in the Appendix.

For more information on Pascal's Triangle and its patterns, go to the following website:

http://ptri1.tripod.com

You may find the hockey stick pattern in Pascal's Triangle interesting.

Conclusion

In this lesson you used tables to help identify and record patterns in shapes and numbers. You investigated how pairs or sets of numbers are related to one another.

People have been interested in patterns for thousands of years. Moreover, this interest in patterns is not restricted to any one culture.

How do you use patterns in your daily life? The girl in this photograph is working on a knitting project that involves a pattern.

BRYAN AND CHERRY ALEXANDER PHOTOGRAPHY

Turn to Assignment Booklet 2A and complete the Lesson 1 Assignment.

Keep Assignment Booklet 2A until you have completed the entire booklet.

Lesson 2

2-D and 3-D Patterns

Patterns are all around you in the everyday world. For example, the bridge in this photograph is made up of a repeated pattern of triangles.

Recognizing, building, and predicting a pattern are important skills and, like all skills, these skills improve with practice.

In this lesson you will use objects like toothpicks, tiles, and cubes to explore **two-dimensional** (2-D) and **three-dimensional** (3-D) **growth patterns**.

Activity 1

Today you will explore growing patterns.

Meeker came running back to join the rest of the crew on the beach. He was clutching a fragrant bouquet of flowers that he had found growing up the slope of a mountain. Once dried and ground into powder, they made a most delicious spice!
I enjoyed Meeker's discovery so much that I named the mountain after him.
- Captain Quinn

Meeker noted a pattern: the higher up the mountain the flowers grew, the larger and more fragrant the flowers became.

Turn to page 17 of your textbook. Look at the three flower designs and the two T-tables that are given.

1. Describe the growing pattern of triangles (green petals) in the flower designs. Then copy and complete the T-table that tells how many triangles (green petals) there are in each design number.

2. Describe the growing pattern of parallelograms (brown leaves) in the flower designs. Then copy and complete the T-table that tells how many parallelograms (brown leaves) there are in each design number.

3. Draw the flower for Design Number 4.

Check your answers on pages 89 and 90 in the Appendix.

Activity 2

Today you will extend 2-D and 3-D patterns by drawing pictures and building models.

For questions 1 to 3, you will need the square tiles from a set of pattern blocks.

1. Turn to page 12 of your textbook. Look at the tiles representing diving platforms in Growth Patterns and Relationships.

   a. Use the square tiles to build the next three higher diving platforms. Draw pictures to show your work.

b. Describe the growing pattern of diving platforms. Then copy and complete the T-table that relates the height of the diving platform to the number of tiles needed in all.

2. Turn to page 13 of your textbook. Look at the tiles that represent gates.

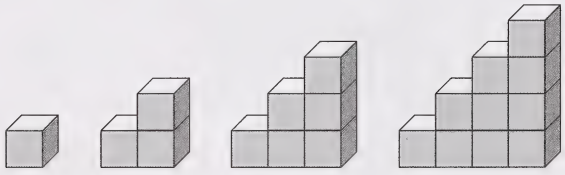
a. Use your square tiles to build the next two larger gates. Draw pictures to show your work.

b. Describe the growing pattern of gates. Then copy and complete the T-table that relates the number of gates to the number of tiles needed in all.

3. Turn to page 2 in the Practice and Homework Book and complete questions 1 and 2.

Check your answers on pages 90 to 92 in the Appendix.

The following pictures show different sizes of staircases made of cubes.



4. a. Use the small cubes from your set of base ten blocks to build the staircases. Then explain how the pattern grows.

b. Develop a T-table to help you predict how many cubes would be needed to build each of the next three staircases.

c. Predict the number of cubes needed to build staircase number 10. Explain how you got your answer.

Check your answers on page 92 in the Appendix.

Activity 3

Today you will build models and draw pictures from pattern descriptions.

For this activity you will need toothpicks and the square tiles from a set of pattern blocks.

1. Trevor used tiles to build a growing pattern for the letter T. He used 5 tiles to build the first T-design. He used 3 more tiles for each larger T-design in the pattern.

   a. Use the description to build the first four T-designs with tiles. Draw pictures to show your models.

   b. Describe how the T-designs grow in two dimensions (for example, width and height).

2. Hannah used 7 tiles to build a growing pattern for the letter H. She used 5 more tiles for each larger H-design in the pattern.

   a. Use the description to build the first four H-designs with tiles. Draw pictures to show your models.

   b. Describe how the H-designs grow in two dimensions.

3. Daiyo made some toothpick designs and developed the following table to record the number of toothpicks used for each design.

Design Number	1	2	3	4	5	6
Number of Toothpicks Used	6	11	16			

   a. Use the table to build the first three designs with toothpicks. Draw pictures to show your work.

   b. Copy and complete the table to predict the number of toothpicks used in the next three designs in the pattern. Then build the designs to check your answers.

c. Describe how the toothpick designs grow.

d. Write a rule to show how the number of toothpicks used is related to the design number.

e. Use your rule to find the number of toothpicks you will use to build design number 25.

Check your answers on pages 93 and 94 in the Appendix.

Sharing Time

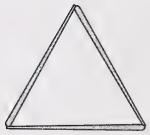
Now it's time to demonstrate what you have been learning to your home instructor.

Turn to page 3 of the Practice and Homework Book and complete questions 1 and 2.

Your home instructor will help you check your answers.

Challenge Activity

The picture at the right shows how you can make a triangle with 3 toothpicks by touching their ends.



1. Use 12 toothpicks to make 6 triangles of the same size and shape. Draw a picture to show what you did.

2. Turn to page 15 of the textbook and answer the problems in questions 1 and 2.

Check your answers on pages 95 and 96 in the Appendix.

Conclusion

In this lesson you used objects like toothpicks, tiles, and cubes to explore 2-D and 3-D growth patterns. You recorded the patterns in tables and found rules that you could use to extend the patterns.

Patterns are all around you. For example, the geodesic dome in this photograph uses a pattern of triangles.

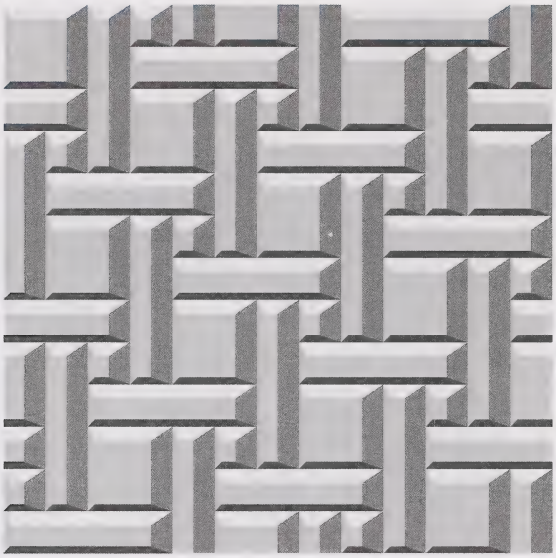
What are other examples that use patterns in the everyday world?

Turn to Assignment Booklet 2A and complete the Lesson 2 Assignment.

Keep Assignment Booklet 2A until you have completed the entire booklet.

Lesson 3

Using Patterns to Solve Problems



Patterns can take many forms. So far in this module, you have examined number patterns, 2-D patterns, and 3-D patterns.

One of the most important reasons for investigating and discovering patterns is so you can use them to solve problems. Although there may be many strategies you can use to solve a problem, finding and applying a pattern can be a timesaving strategy.

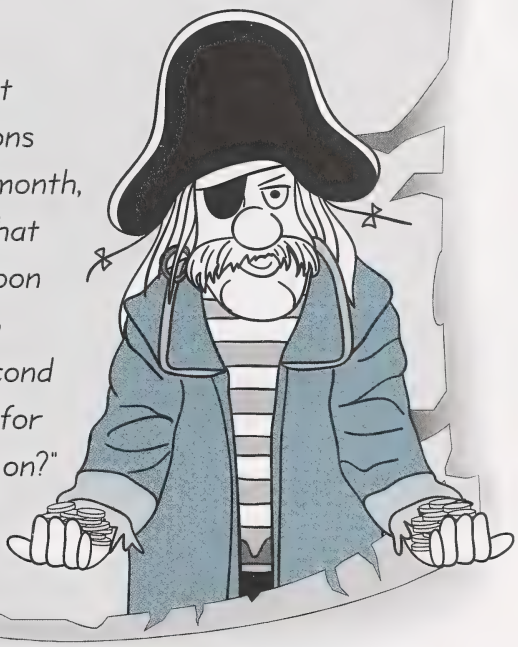
In this lesson you will focus on exploring logic patterns and using patterns to solve problems. You will use special diagrams (called Carroll diagrams and Venn diagrams) to help you solve problems.

Activity 1

Today you will use patterns to solve problems.

At sea, the crew members often must work every day of the month. When payday for the month of September arrived, I offered the cook a choice.

"Would ye prefer that I gave ye ten doubloons for each day of this month, or would ye prefer that I gave you one doubloon for the first day, two doubloons for the second day, three doubloons for the third day, and so on?"
- Captain Quinn



Which of the Captain's methods for getting paid do you think the cook should choose? One way to decide would be to make tables and look for patterns to compare the two methods of payment.

1. Copy and complete the following tables.

Method A

Day of the Month	1	2	3	4	5	6	7	8	9	10
Doubloons Received for the Day	10	10	10							
Total Doubloons	10	20	30							

Method B

Day of the Month	1	2	3	4	5	6	7	8	9	10
Doubloons Received for the Day	1	2	3							
Total Doubloons	1	3	6							

2. **a.** Describe how the total number of doubloons grows for Method A.
   b. How does the total number of doubloons grow for Method B?

3. **a.** For Method A, write a rule that relates the total number of doubloons received to the day of the month.

   b. For Method B, write a rule that relates the total number of doubloons received to the day of the month.

4. **a.** Use the rules in question 3 to predict the total amount received in September for Method A.

   b. Predict the total amount for Method B.

5. Which method of payment (Method A or Method B) should the cook choose? How much greater would the pay be using that method?

Check your answers on pages 96 and 97 in the Appendix.

Are you ready for some more growing patterns?

6. Turn to page 31 of your textbook. Do questions 1 to 4 of On Your Own. Make tables and look for patterns to help you solve the problems.

Check your answers on pages 98 and 99 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Turn to page 15 of the Practice and Homework Book and answer question 1. You may use your calculator.

Your home instructor will help you check your answers.

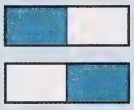
Activity 2

Today you will use tables to find all possibilities.

Looking at the windows shown below, Kassidy said she could see 3 rectangles in the top window and 6 rectangles in the bottom window. Connor was puzzled because he could only see 2 rectangles in the top window and 3 rectangles in the bottom window.

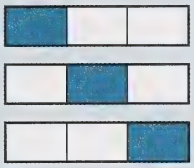
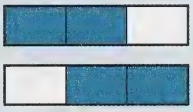
Kassidy showed Connor how a table can be helpful in a situation like this to make an organized list of all possible solutions. She made the following two tables.

This table shows the total number of rectangles in the top window.

Size of Rectangle	Rectangles of That Size	Number of That Size
		2
		1

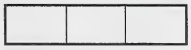
Total number of rectangles = 3

This table shows the total number of rectangles in the bottom window.

Size of Rectangle	Rectangles of That Size	Number of That Size
		3
		2
		1

Total number of rectangles = 6

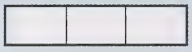
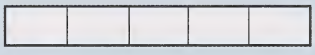
1. Copy and complete the following table for a window that has 4 panes. Then find the total number of rectangles in the window.

Size of Rectangle	Rectangles of That Size	Number of That Size
		
		
		
		

Check your answers on page 99 in the Appendix.

Connor liked how the tables helped to find all the rectangles. He got even more excited when he realized that he could use the pattern to predict the number of rectangles in windows without having to draw all the rectangles.

Connor made the following table to show how the total number of rectangles in a window of this shape is related to the number of panes.

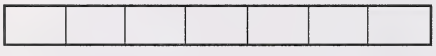
Window	Number of Panes in the Window	Total Number of Rectangles
	2	2 + 1 = 3
	3	3 + 2 + 1 = 6
	4	4 + 3 + 2 + 1 = 10
	5	5 + 4 + 3 + 2 + 1 = 15

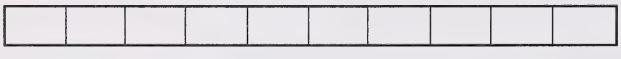
2. Confirm that a window with 5 panes has 15 rectangles by drawing pictures that show all the possible rectangles.

3. Describe the pattern that tells how the total number of rectangles grows each time the number of panes in the windows increases by one.

4. Write a rule that relates the number of panes in any window of this type to the total number of rectangles that can be seen.

5. Use your rule to find the total number of rectangles in each of the following windows.

   a. 

   b. 

Check your answers on pages 100 and 101 in the Appendix.

Claire's grandpa is building play furniture for Claire and her cousins. He has 31 legs to use and he wants to make chairs and stools. The chairs have 4 legs each and the stools have 3 legs each. How many chairs and how many stools could he make with the 31 legs?

To solve this problem, you could use the guess-and-test method. However, if you want to find **all** the possible numbers of chairs and stools, you may find it more helpful to make a table.

6. Copy and complete the following table.

Number of Chairs Made	Number of Legs Used for Chairs	Number of Legs Left Over	Number of Stools That Can Be Made with Leftover Legs	Number of Legs Left Over	Are all 31 legs used?
1	$1 \times 4 = 4$	$31 - 4 = 27$	$27 \div 3 = 9$	0	yes
2	$2 \times 4 = 8$	$31 - 8 = 23$	$23 \div 3 = 7$ R2	2	no
3					
4					
5					
6					
7					

7. Could Claire's grandpa possibly make more than 7 chairs? Explain.

8. If all the legs are used, what possible combinations of chairs and stools can he make?

9. Make another table that is similar to the table you made in question 6, except find the number of legs used to make stools first. Then find how many chairs can be made.

Check your answers on pages 101 and 102 in the Appendix.

Activity 3

Today you will use logical reasoning.

Connor put gifts for three friends, Bob, Shirley, and Percy, into identical boxes. He wrapped each box with a different colour of paper: purple, blue, and silver.

After he had the gifts wrapped, Connor forgot whose gift he had put into each box. He did, however, remember that no one's gift was wrapped in a colour that starts with the same letter as his or her name. He also remembered that he didn't use purple for Shirley's gift because she doesn't like that colour.

You can use these clues and a **Carroll diagram** to help identify the gifts without having to open the boxes. The Carroll diagram matches each person's name with each colour of paper.

Work through the following steps to see how you can use logical reasoning to solve the problem.

Step 1: No one's gift was wrapped in a colour that starts with the same letter as his or her name, so you can write **no** in the cells that match those names and colours.

	Bob	Shirley	Percy
Purple			no
Blue	no		
Silver		no	

Step 2: Shirley's gift was not wrapped in purple because she doesn't like that colour. You can write **no** in the cell that matches her name with purple. There is now only one empty cell in Shirley's column. So **yes** must go in the cell that matches her name with blue. This means Percy's gift cannot be blue, so write **no** in the cell that matches Percy and blue.

	Bob	Shirley	Percy
Purple		no	no
Blue	no	yes	no
Silver		no	

Step 3: Since neither Shirley nor Percy have purple paper, you know that Bob's gift must be wrapped in purple. You can write **yes** in the cell that matches his name with purple. This means that Bob's gift cannot be silver, so write **no** in the cell that matches Bob and silver.

	Bob	Shirley	Percy
Purple	yes	no	no
Blue	no	yes	no
Silver	no	no	

Step 4: Percy's gift must be wrapped in silver because it is the only colour left. You can write **yes** in the cell that matches his name with silver.

	Bob	Shirley	Percy
Purple	yes	no	no
Blue	no	yes	no
Silver	no	no	yes

The gift wrapped in purple is Bob's. The gift wrapped in blue is Shirley's. The gift wrapped in silver is Percy's.

Mark, Samuel, Anna, and Celine went to the ice-cream shop to buy their favourite cones.

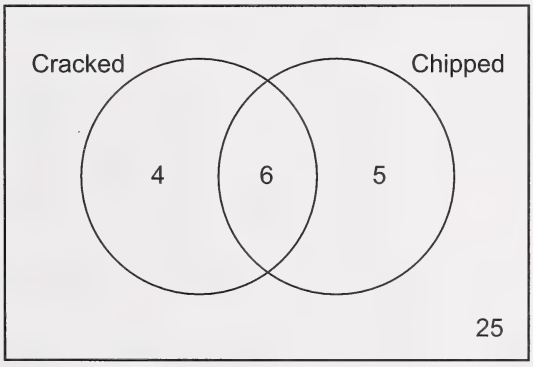
- Each person prefers a different flavour of ice cream: chocolate, mint, strawberry, or vanilla.
- No one prefers a flavour with the same number of letters as his or her name.
- The person who loves vanilla always puts chocolate on top of the ice cream.
- Anna is allergic to chocolate.
- Samuel says vanilla is boring.
- The girl who prefers mint ice cream likes her ice cream in sugar cones.

1. Match each person with his or her favourite ice cream. A good strategy is to use a Carroll diagram like the one used in the problem about the gifts.

Check your answers on pages 102 and 103 in the Appendix.

Reilly bought some plates at a garage sale. He used two hoops to sort the plates. Some plates were both chipped and cracked, so he overlapped the hoops and put the plates that were both chipped and cracked in the intersection. He stacked the plates that were only cracked in the outer part of the left hoop. He stacked the plates that were only chipped in the outer part of the right hoop. He stacked all the plates that were neither chipped nor cracked outside the hoops.

Reilly then drew the following **Venn diagram** to describe the plates.



2. Use the Venn diagram to answer the following questions.

   a. How many of the plates were both cracked and chipped?
   b. How many of the plates were only cracked?
   c. How many of the plates were only chipped?
   d. How many of the plates were not cracked or chipped?
   e. How many plates did Reilly buy in total?

Check your answers on page 103 in the Appendix.

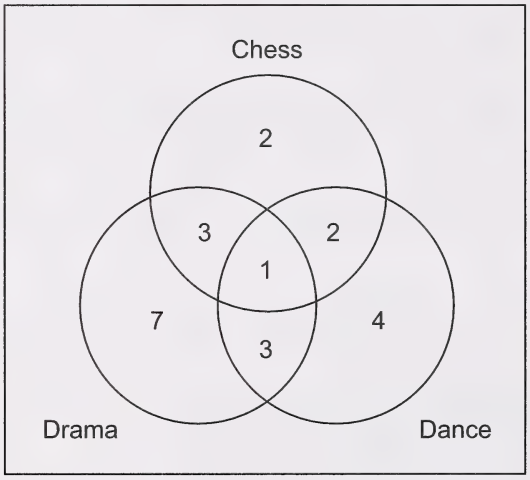
WE NEED YOU!

Needed:

Chess Club Members
Drama Club Members
Dance Club Members

If you are interested in joining any of these clubs,
call Mr. Rasbit at **555-9876**.

Mr. Rasbit used the following Venn diagram to sort the students who signed up for the clubs.



3. Use the Venn diagram to answer the following questions.

a. How many people chose dance?
b. How many people chose chess?
c. How many people chose drama?
d. How many people didn't choose dance?
e. What is the total number of people that signed up for the clubs?

Check your answers on page 103 in the Appendix.

Challenge Activity

Judge Owen Owl has to decide which character is guilty of spilling the milk. He took the following statements:

Bruce Bull: "Bobby Bear spilled the milk."
Digger Dog: "Bobby Bear is lying."
Bobby Bear: "Digger Dog spilled the milk."
Randy Rooster: "I didn't spill the milk."

Only one of the characters is telling the truth. Only one of the characters is guilty.

1. Who is telling the truth? Who spilled the milk?

Check your answers on page 104 in the Appendix.

Detective Beagle is investigating a robbery in the forest. He takes the following statements:

Betty Beaver: Cheryl Chipmunk did it.
Cheryl Chipmunk: I didn't do it.
Sandy Squirrel: I didn't do it.

Detective Beagle knows that only one of the characters is telling the truth. He also knows that only one of the three is guilty.

2. Who is telling the truth? Who is guilty?

Check your answers on page 105 in the Appendix.

Conclusion

In this lesson you focused on exploring logic patterns and using patterns to solve problems. You used special diagrams called Carroll diagrams and Venn diagrams to help you solve problems.

Carroll diagrams and Venn diagrams are each named after the mathematician who invented them. Carroll diagrams are named for Lewis Carroll, the pen name for Charles Dodgson (1832–1898). Venn diagrams are named after John Venn (1834–1923)

Did you know that Lewis Carroll also wrote the books *Alice in Wonderland* and *Through the Looking Glass*?

Turn to Assignment Booklet 2A and complete the Lesson 3 Assignment.

When you are done, send Assignment Booklet 2A to your distance learning teacher to be marked.

Lesson 4

Multiples and Factors

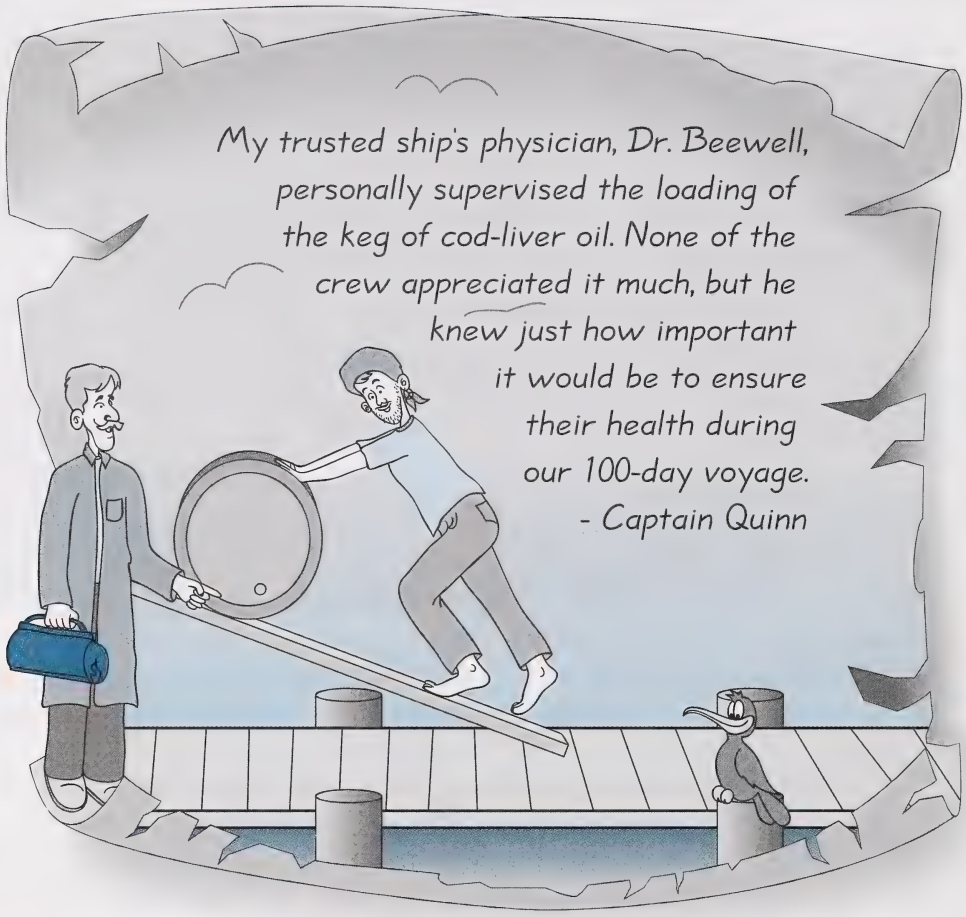
In earlier grades you used skip counting to solve some problems. For example, you counted by 5s to find the value of a handful of nickels.

When you were counting by 5s, you were finding multiples of 5.

In this lesson you will use patterns to explore multiples and factors. You will discover that the arrays you used for multiplication can show how multiples and factors are related. You will also use tables to find patterns in factor pairs.

Activity 1

Today you will explore multiples.



Dr. Beewell administered one spoonful of cod-liver oil to each crew member every four days, beginning on the morning of the fourth day of a voyage. He used a chart to keep track of the days he gave the medicine.

1. Use the hundred chart from the Cut-Out Learning Aids section of the Appendix and counters to show the days on which the cod-liver oil was given. Beginning on 4, place a counter on every fourth number on the hundred chart until you see a visual pattern being formed. **Note:** You can use dried beans, buttons, or bread tags for counters.

   a. Continue the pattern using the counters.

   b. The numbers covered by the counters are called the multiples of 4. List them.

Now use the constant addend feature of your calculator to verify your list of the multiples of 4.

Keystrokes	ON/C	4	+	4	=	=	=	=	=	=	=	=	=	=
Display	0	4	4	4	8	12	16	20	24	28	32	36	40	44

   c. How many times will each sailor get cod-liver oil on a 100-day voyage?

Check your answers on page 106 in the Appendix.

Dr. Beewell wanted to plan for the next voyage, which would last 200 days.

2. **a.** Use addition to describe how the multiples of 4 grow.

   b. Use addition to find the next five days, after the hundredth day, that cod-liver oil will be given.

3. **a.** Use multiplication to describe how this number pattern grows.

   b. Use multiplication to find the next five days, after the hundredth day, that cod-liver oil will be given.

4. **a.** Explain how you can use division to decide whether a number is a multiple of 4.

   b. Use division to predict whether cod-liver oil will be given on days 112, 146, and 160.

5. For the first 100 callers each week on the Kids Carnival TV show, every sixth caller, starting with the sixth caller, wins a movie pass.

   a. Which callers will win a movie pass?

   b. If the contest is extended past 100 calls, which callers will be the next five to win movie passes? Show your work.

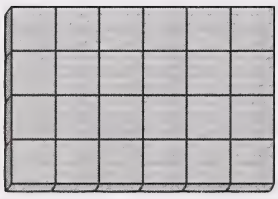
   c. Use division to predict whether callers 138, 152, and 198 will win passes if the contest is extended past 100 calls. Show your work.

Check your answers on pages 106 and 107 in the Appendix.

Activity 2

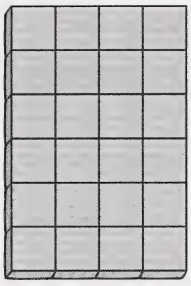
Today you will explore multiples and factors.

Barry placed squares of fudge on a tray as shown.



You can see that there are 4 rows on the tray, and there are 6 fudge squares in each row, so there are $4 \times 6 = 24$ fudge squares on the tray.

If Barry turned the tray as shown, there are still 24 fudge squares, but now you can see 6 rows with 4 fudge squares in each row. There are $6 \times 4 = 24$ fudge squares.



Now, use these arrays to help you divide.

- If 24 fudge squares are to be shared equally among 4 people, you can see that each person will get 6 squares.
- If 24 fudge squares are to be shared equally among 6 people, you can see that each person will get 4 squares.

The numbers 4 and 6 are called the **factors** of 24 because 24 can be divided evenly by 4 and 6. The number 24 is called a **multiple** of 4 and 6.

You can use the same array of 24 squares to show 4 rows of 6 squares or 6 rows of 4 squares. For this reason, 4 and 6 are called a **factor pair** for 24.

You can find other factor pairs for 24 by making different rectangular arrays with 24 square tiles. The number of rows will represent one factor in the pair and the number of tiles in each row will represent the other factor in the pair.

1. Use the square tiles in a set of pattern blocks to find all the other rectangular arrays Barry could have made with the 24 fudge squares. You only need to make one array for each factor pair. Then draw pictures of the arrays you have made and label them to show all the factor pairs of 24.

2. Explain how you could use tiles to find all the factors of any given number.

3. **a.** Use the square tiles in a set of pattern blocks to find all the factors of 16. Then draw diagrams to show all your arrays.

   b. List all the factors of 16 from least to greatest.

4. Take a handful of tiles. Make all the possible arrays with your set of tiles. Try doing this several more times with different sets of tiles. Is the number of possible arrays the same for every set of tiles?

5. What is the least number of arrays that can be made with any set of tiles? Explain.

6. Explain how can you use an array to find the total number of tiles in any set.

Check your answers on pages 108 and 109 in the Appendix.

Activity 3

Today you will use charts to find factors.

In Activity 2 you learned how to find all the factors of a number by making arrays with tiles. That is a good method because you can actually see all of the factors. Now, you will look for patterns that will help you find factors of a number without using tiles.

To begin with, you may find it helpful to visualize placing tiles in rows. Each time you make an array, you can find a pair of factors. Remember that a factor of a number is any number that divides into it evenly.

You know that you can always make an array by placing all the tiles in a single row. This shows that the number 1 is a factor of every number, and every number is a factor of itself.

1. If you know one of the factors in a factor pair, how do you find the other factor?

2. How can you predict if the number 2 is a factor of a given number?

Check your answers on page 109 in the Appendix.

To find all the factors of a number, it is helpful to use an organized approach. If you look for the factors in order of size, from least to greatest, then you will not miss any of them.

Example

The following table can be used to find all the factors of 18.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
1	18 ÷ 1 = 18	yes	18
2	18 ÷ 2 = 9	yes	9
3	18 ÷ 3 = 6	yes	6
4	18 ÷ 4 = 4 R2	no	—
5	18 ÷ 5 = 3 R3	no	—
6	18 ÷ 6 = 3	yes	3

Because the factors come in pairs, you can stop looking for more factors when the factor pairs start to repeat.

The factors of 18 are 1, 2, 3, 6, 9, and 18.

You can also use a T-table to find all the factors of a number.

Example

The following T-table can be used to find all the factors of 18.

18	
1	18
2	9
3	6
~~4~~	← 4 is not a factor of 18.
~~5~~	← 5 is not a factor of 18.
~~6~~	~~3~~ ← The factors are starting to repeat.

Notice that the numbers that are not factors and the repeated factors are crossed out.

Read down the left column of the T-table and read up the right column to list the factors in order from least to greatest.

The factors of 18 are 1, 2, 3, 6, 9, and 18.

3. **a.** Use an organized approach to find all the factors of 30.
b. List the factors of 30 from least to greatest.

4. Turn to page 67 of your textbook. Do questions 1 to 4 of On Your Own.

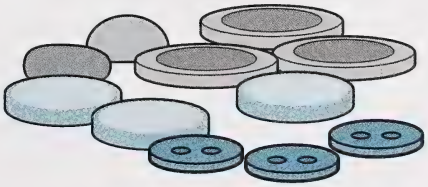
Check your answers on pages 109 to 111 in the Appendix.

Sharing Time

Now it's time to show your home instructor what you have been learning.

Turn to page 27 of the Practice and Homework Book. Play the game with your home instructor.

Instead of crossing out numbers on the game board, cover the numbers with counters. You will need two kinds of counters, one kind for yourself and one kind for your home instructor. You could use two kinds of dried beans, red and black checkers, or two kinds of coins.



Challenge Activity

1. Some pirates were trying to share three sacks of doubloons.

   - The first sack could be shared equally among 2 pirates, but not 3.
   - The second sack could be shared equally among 4 pirates, but not 5.
   - The third sack could be shared equally among 3 pirates, but not 4.

   If the coins from the three sacks were combined, there would be less than 35 doubloons in all, and they could be shared equally among 5 pirates. How many doubloons may have been in each sack?

2. Turn to page 44 of the Practice and Homework Book and answer questions 1 to 4.

Check your answers on pages 111 and 112 in the Appendix.

Conclusion

In this lesson you discovered how factors and multiples are related to multiplication and division. You also saw that finding factors and multiples can help you to solve problems.

Learning mathematics is similar to building a house. If you have a strong foundation of knowledge, it is easier to master new concepts and skills.

Turn to Assignment Booklet 2B and complete the Lesson 4 Assignment.

Keep Assignment Booklet 2B until you have completed the entire booklet.

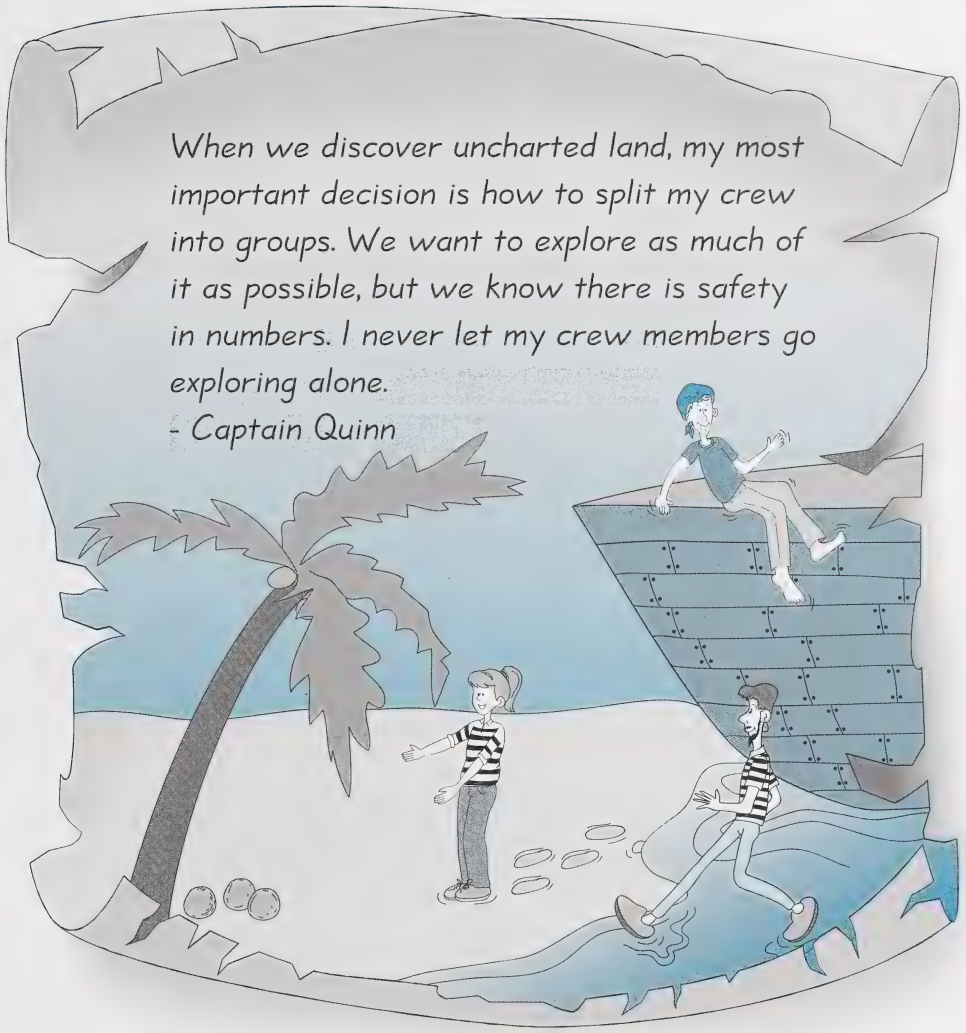
Lesson 5

Composite and Prime Numbers

In Lesson 4 you learned about factors and multiples. In this lesson y ou will see how you can use the patterns you found in arrays and factor pairs to identify composite numbers and prime numbers. You will also use your understanding of these special sets of numbers to solve problems.

Activity 1

Today you will work with composite and prime numbers.



Once Captain Quinn knew how many of his crew were available to form an exploration team, he had to decide how to split them up into smaller groups. He wondered if he should make several small groups or fewer large groups. More groups meant that more territory could be explored, but the larger the groups were, the safer they would be.

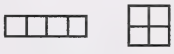
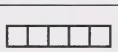
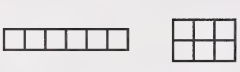
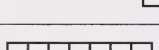
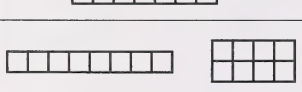
Captain Quinn gave his quartermaster the following rules for splitting the exploration team:

Rule 1: The exploration team had to split into two or more smaller groups. The groups had to be of equal size so that tasks could be divided evenly.

Rule 2: There had to be at least two crew members in each smaller group. In other words, one sailor could not explore alone.

Rule 3: If the exploration team could not split into equal-sized smaller groups of two or more sailors, that team could not go ashore.

The quartermaster made the following table to show which sizes of teams could or could not split up according to Captain Quinn's rules. He wanted to list all the different ways that exploration teams could split into smaller groups. To do this, the quartermaster visualized teams of sailors standing in arrays so that rows of sailors could form smaller groups of equal size. He used small cubes to represent teams of sailors and made all the possible arrays for each team.

Number of Sailors Available to Go Exploring	Possible Array(s) of Sailors That Can Be Made	Can that size of team go ashore? If so, what smaller groups can be made?	Factor Pairs Shown by the Arrays
1		No	1×1
2		No	1×2
3		No	1×3
4		Yes. 2 groups of 2.	1×4, 2×2
5		No	1×5
6		Yes. 2 groups of 3 or 3 groups of 2	1×6, 2×3
7		No	1×7
8		Yes. 2 groups of 4 or 4 groups of 2.	1×8, 2×4

After he had completed the table for up to 8 sailors, the quartermaster thought he saw a pattern. He predicted that an exploration team could go ashore only if it had an even number of sailors.

1. Use the small cubes from a set of base ten blocks to make arrays to represent teams of 9, 10, 11, and 12 sailors.

   a. Extend the quartermaster's table to record your results.

   b. Does the quartermaster's prediction always work? Explain why or why not.

2. **a.** For which numbers of sailors could you make only a single row? In other words, what size of team cannot go ashore?

   b. How many factors do these numbers have?

3. **a.** For which numbers of sailors could you make both a single row and another array? In other words, what size of team can go ashore?

   b. How many factors do these numbers have?

Check your answers on page 113 in the Appendix.

The arrays of sailors are related to **prime** and **composite** numbers.

A prime number has exactly two different factors (the number itself and 1). A composite number has more than two different factors.

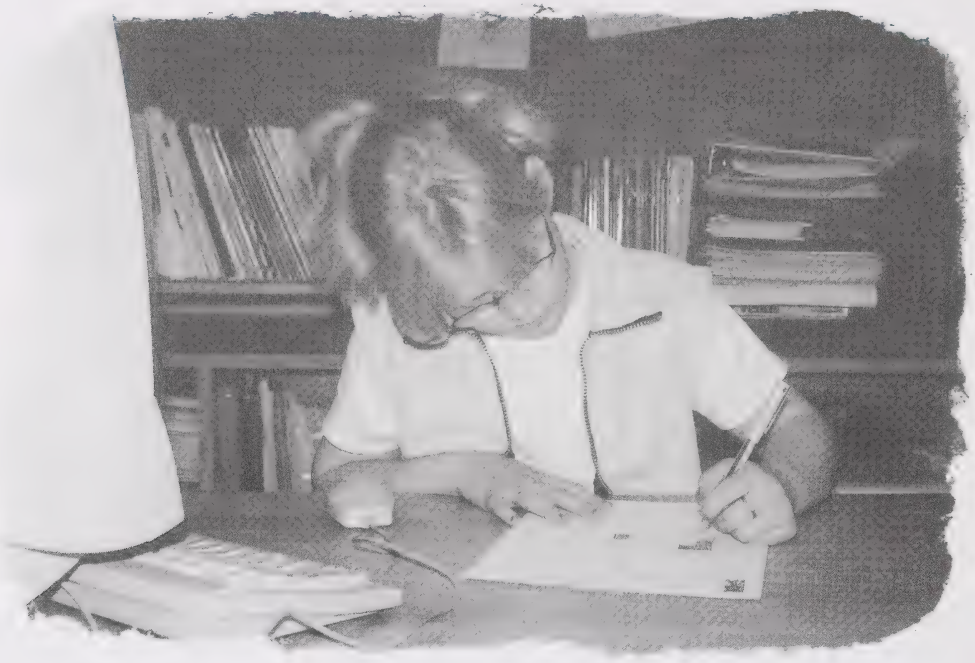
4. Explain why the number 1 is neither prime nor composite.

5. Explain why the number 2 is the only prime number that is even.

Check your answers on page 113 in the Appendix.

Activity 2

Today you will use grid paper to decide if a number is prime or composite.

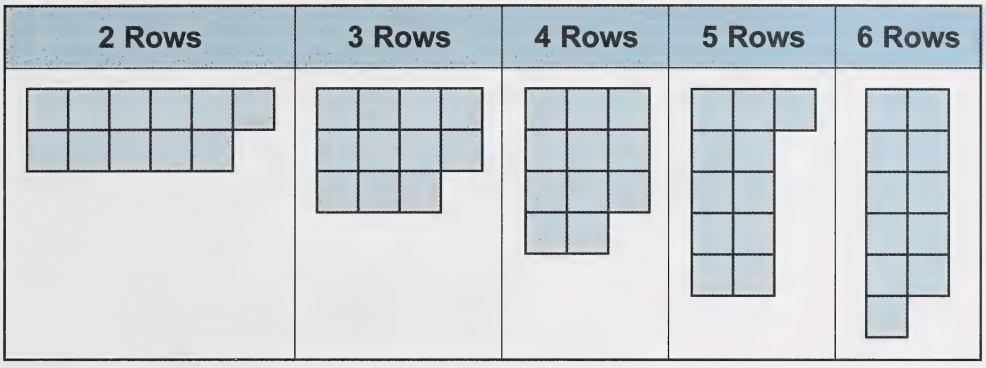


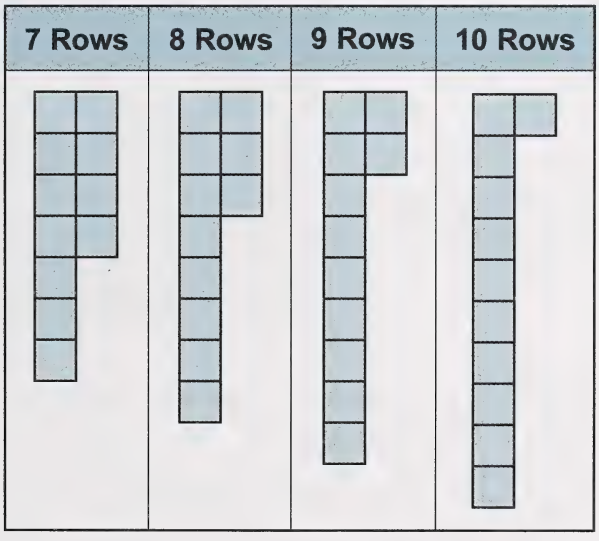
Alka said, "Name any whole number greater than 1, and I can use grid paper to show if it's prime. If the only rectangle I can shade on the grid for the number has all the squares in a single line, then the number is prime. Otherwise, it is composite."

1. Use grid paper to show all the rectangles Alka would shade to show that 24 is a composite number.

Check your answers on page 114 in the Appendix.

Alka found it easier to decide about some numbers than others. For example, when she was trying to decide if 11 is prime or composite, she used an organized approach. First, she tried to shade two rows of squares, then three rows, and so on, until she was sure that she could not shade any rectangles. She decided that 11 is prime. Her work is shown below.





2. Use Alka's method of shading rectangles to decide if the following numbers are prime or composite.

a. 13
b. 15

3. Chin looked at the drawings that Alka used to decide that 11 was prime. He said that after she tried to make a rectangle with six rows, she did not have to try to make any more rectangles. Do you agree with Chin? Explain.

4. If Alka had used Chin's rule when she tried to shade rectangles to decide if 13 was prime, how many rows would she have tried? Explain.

5. Use Chin's rule to decide if 17 is prime or composite.

Check your answers on page 114 to 116 in the Appendix.

Mandy used an organized approach similar to Chin's, but she did not actually shade rectangles. She said, "I use division to find factor pairs for a number where both factors are greater than 1. When I find a pair like that, I can picture an array and I know that the number is composite. When I can't find a pair like that, I know that the number is prime."

Mandy made the following chart to decide if 11 is prime.

Factor Tried (numbers of rows in array)	Test it using division.	Is it a factor?	What is the other factor in the pair?
2	$11 \div 2 = 5$ R1	no	—
3	$11 \div 3 = 3$ R2	no	—
4	$11 \div 4 = 2$ R3	no	—
5	$11 \div 5 = 2$ R1	no	—
6	$11 \div 6 = 1$ R5	no	—

6. Mandy said she can tell from her chart that if she tries to make six or more rows, she can't put two or more squares in each row. Explain how she knows this.

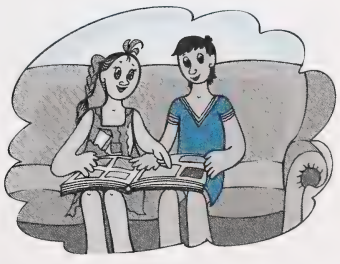
7. Make charts like Mandy's to show how she would decide if 13 and 17 are prime. Explain how you know when you don't have to try more rows.

Check your answers on pages 116 and 117 in the Appendix.

Activity 3

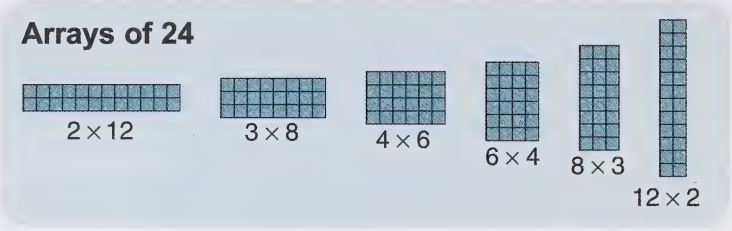
Today you will use a pattern to solve problems with composite and prime numbers.

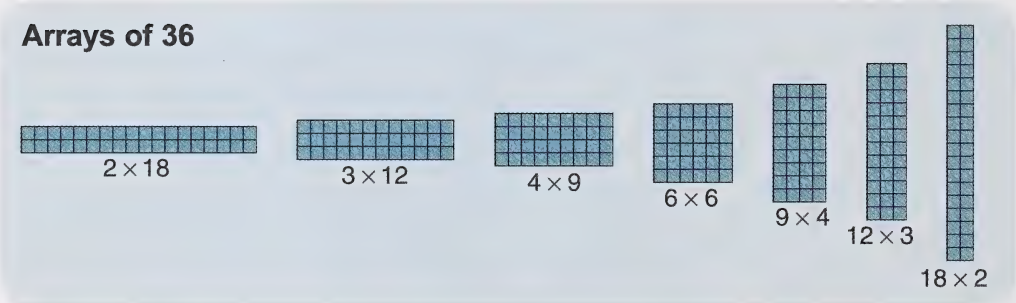
Rhadia had 17 photos that she wanted to put in a photo album. She thought they would look best in a rectangular array, but she wasn't sure if it was possible.



Rhadia remembered using division to find all the factors of a composite number. She remembered that she only had to test half of the factors because they came in pairs. When the factors in a pair were switched, the array was just turned sideways.

The following diagrams show all the arrays for the composite numbers 24 and 36. When looking at arrays, the multiplication sign is read "by." For example, 2×12 is read "two by twelve."



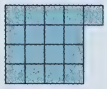


Rhadia noticed that as you make more rows, the array gets closer to forming a square. Once the array becomes a square, or as close to a square as possible, the factor pairs repeat. Rhadia decided to use this idea to decide if 17 is prime or composite. She first tried to make a square with her photos.

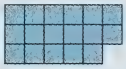
"If I can't make a square," Rhadia said, "I will rearrange the photos and make arrays with one less row each time. If I can't make a rectangle before I get to one long row, then I'll know that 17 is a prime number."

Rhadia's work is shown below:

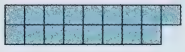
"The closest I could get to forming a square shape was by making 4 rows with 4 photos, but there was 1 extra photo, so 4 rows didn't work."



"Next, I tried 3 rows of 5 photos, but there were 2 extra photos, so 3 rows didn't work."



"Next, I tried 2 rows of 8 photos, but there was 1 extra photo, so 2 rows didn't work. I knew that meant that 17 is prime."



Since 17 is prime, Rhadia knew she couldn't put her photos in a rectangular array. Rhadia also remembered the tables Mandy made in Activity 2 to decide if a number was prime. Rhadia and Mandy compared the steps they took.

Mandy noticed that the more rows she made, the less squares were in each row. When she got to the point where the number of squares in each row was the same as or less than the number of rows, she really didn't need to try any more factors in her table.

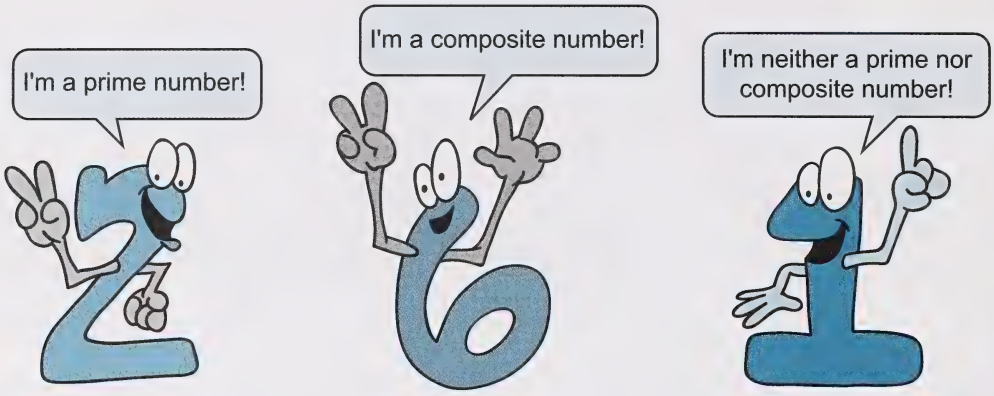
Mandy said, "I started with two rows and worked up to a square. Rhadia started with a square and worked down to two rows. That's neat!"

1. Shade squares to show the arrays Rhadia would try to make to decide whether 19 and 31 are prime or composite.

2. Make tables like Mandy's to decide if 19 and 31 are prime or composite. Explain how Mandy will know when she has tried enough factors.

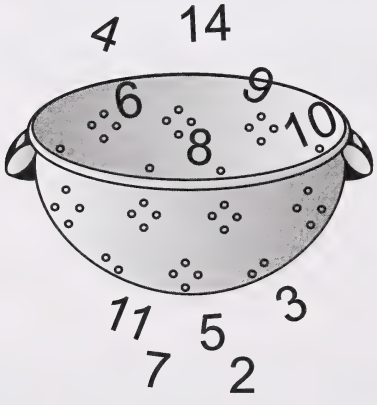
Check your answers on pages 118 and 119 in the Appendix.

Challenge Activity



Eratosthenes (284–192 B.C.), an ancient Greek mathematician, invented a method called the **sieve of Eratosthenes** for separating prime numbers from other numbers.

The sieve of Eratosthenes can be used to sort the prime numbers from as small or large a set of numbers as you like.



The sieve of Eratosthenes uses the following steps:

- Write the set of numbers.
- Circle 2 and cross out all the multiples of 2.
- Circle 3 and cross out all the multiples of 3.

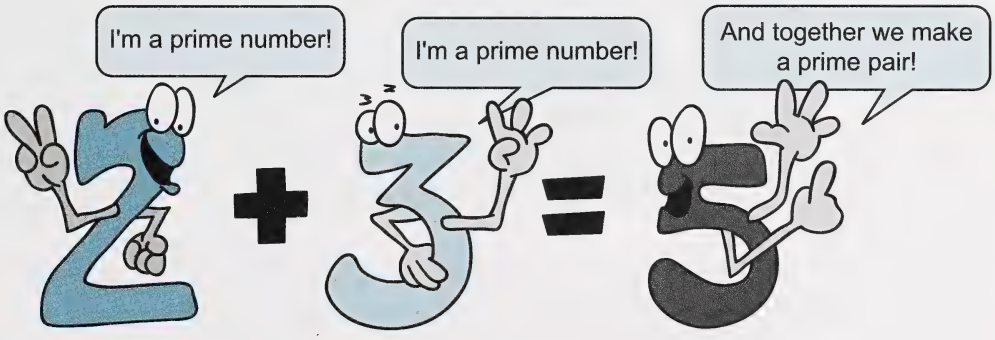
- Circle 5 and cross out all the multiples of 5.
- Circle 7 and cross out all the multiples of 7.
- Circle the remaining numbers.

The circled numbers are the prime numbers. The crossed-out numbers are the composite numbers.

Note: Number 1 is neither circled nor crossed out because it is not a prime number or a composite number.

1. Use the sieve of Eratosthenes to find the prime numbers less than 50.

Check your answers on page 120 in the Appendix.

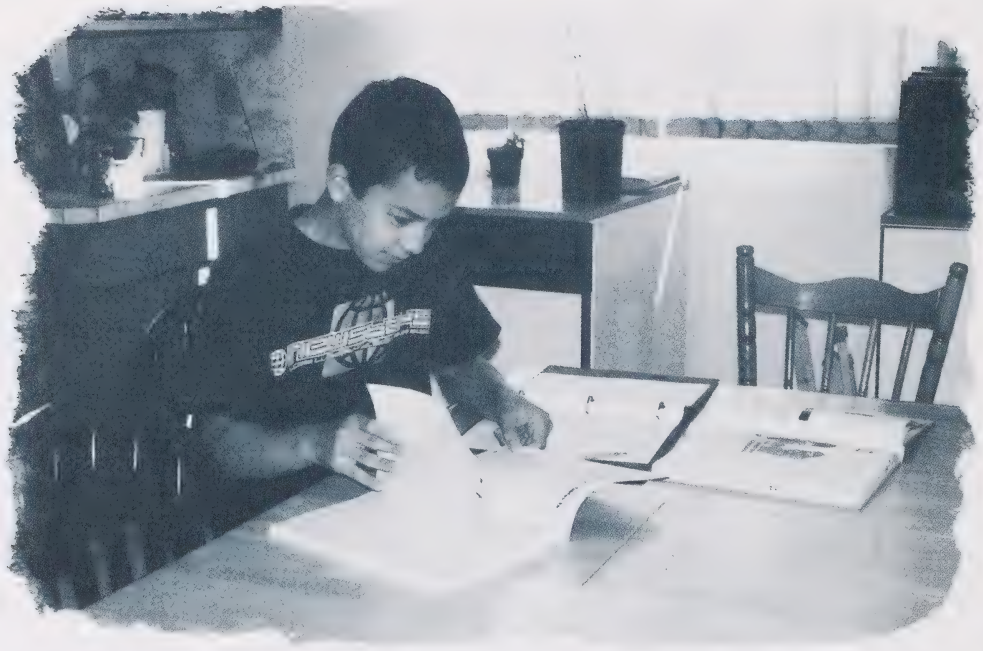


2. Name pairs of prime numbers less than 50 whose sum is a prime number.

3. What is the same about all your prime pairs? Explain.

Check your answers on page 120 in the Appendix.

Conclusion



In this lesson you learned how to decide if a number is prime or composite. You saw how you could solve some practical problems using the patterns you found.

Turn to Assignment Booklet 2B and complete the Lesson 5 Assignment.

Keep Assignment Booklet 2B until you have completed the entire booklet.

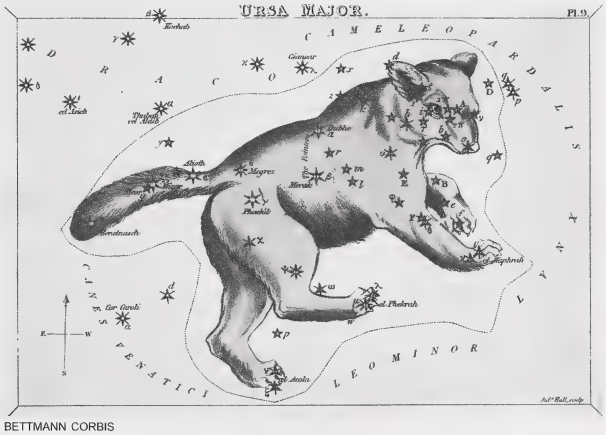
Module Summary

In Module 2 you extended your understanding of patterns. You learned to identify and extend patterns in pictures and numbers, and you built 2-D and 3-D patterns. You learned about some special patterns, such as multiples and factors, and how they are related to composite and prime numbers. You made tables and used patterns to solve problems.

In the Module Overview, you discovered that the favourite star pattern of Captain Quinn and his crew were the Big Dipper and the Little Dipper.

Different cultures have different names for these star groupings. In England, the seven stars that make up the Big Dipper are called the Plough. In Germany, they are known as Charles's Wagon. For the Lakota tribe, which is known for its star knowledge, they are called *Wichakihuyapi* (a dipper or ladle) and sometimes *Ocheti Sakowin* (seven council fires).

The Big Dipper and the Little Dipper are parts of two constellations called Ursa Major (Great Bear) and Ursa Minor (Lesser Bear). Use the Internet to find out more about Ursa Major, Ursa Minor, and other constellations.



BETTMANN CORBIS

Turn to Assignment Booklet 2B and complete the Numbers in the News project.

When you are done, send Assignment Booklet 2B to your distance learning teacher to be marked.

Keystrokes

Take out your calculator and complete the following exercises. They will help you review some of ideas you have learned in Module 2.

Funky Feature: Constant Multipliers

You can enter a multiplier and use it over and over again without re-entering it. The example below shows how once you multiply the first number by 3, you can enter any other number you wish to multiply by 3 and press the = key.

Keystrokes	ON/C	3	×	4	=	5	=	8	=	10	=	100	=
Display	0	3	3	4	12	5	15	8	24	10	30	100	300

1. a. Enter 101 as a constant multiplier and use it to find the first three products.

- 101×23
- 101×45
- 101×67

b. Find a pattern you can use to predict the product of 101 and any 2-digit number.

c. Enter any 2-digit numbers you wish, predict the products when you multiply these numbers by 101, and then check your predictions on the calculator.

2. a. Enter 37 037 as a constant multiplier and use it to find the first three products.

- $37\,037 \times 3$
- $37\,037 \times 6$
- $37\,037 \times 9$

b. Find a pattern, predict the next three products, and then check your predictions.

- 37 037 × 12
- 37 037 × 15
- 37 037 × 18

Experiment and see if you can find some more funky multipliers.

Check your answers on pages 120 and 121 in the Appendix.

Funky Feature: Count On It!

Your calculator can help you keep your mental computation skills sharp. The following example shows how you can use your calculator to find multiples of a number.

Example

Keystrokes	ON/C	5	+	5	=	=	=	=	=	=	=	=	=	=
Display	0	5	5	5	10	15	20	25	30	35	40	45	50	55

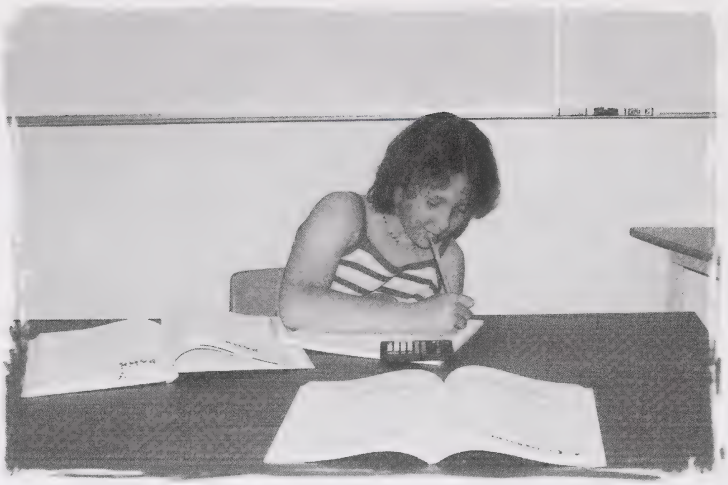
3. a. You may find counting by 2s, 5s, or 10s easy, but what about 7s or 23s? Challenge yourself! Enter any number the same way 5 was entered above. Then, each time, say the next number before you press the = key.

b. How about counting multiples backwards? Enter a factor you'd like to use and multiply it by a large number to get a starting multiple. Following is an example.

Keystrokes	ON/C	7	×	51	=	–	7	=	=	=
Display	0	7	7	51	357	357	7	350	343	336

Check your answers on page 121 in the Appendix.

Review



This review will help you apply what you learned in Module 2 and prepare for the final test. Discuss with your home instructor when you should begin the Review and how much of the Review you should complete.

1. Do Skill Bank from This Unit questions 1.a. to 1.d. on page 38 of your textbook. For each question, write the rule that tells how to find the next number, and write the next three numbers in each pattern.

2. Do Skill Bank from This Unit questions 2.a. to 2.d. on page 38. For each question, write the rule that tells what you do to the input numbers to get the output numbers, and copy and complete the table.

3. Do Skill Bank from This Unit questions 4.a. to 4.d. on page 39. For each question, write the rule that tells how to find the next number, and write the next three numbers in each pattern.

4. Do Problem Bank questions 1.a. and 1.b. on page 36.

If you need help with questions 1 to 4, look back at Lesson 1, where you learned about identifying patterns. If you feel you need more practice, do question 5.

5. Do Skill Bank Looking Back question 1 on page 59 of your textbook.

Check your answers on pages 122 to 124 in the Appendix.

6. Look at the growing pattern shown in question 3 on page 39 of your textbook.

a. Describe how the pictures in the pattern grow.

b. Copy the following T-table into your notebook and extend it to include the first 10 designs.

Design Number	Number of Squares in the Design
1	2
2	6
3	12
4	20

c. Explain how the number of squares in each design is related to the design number.

7. Do Problem Bank question 4 on page 37 of your textbook.

If you need help with questions 6 and 7, look back at Lesson 2, where you learned about 2-D and 3-D patterns. If you feel you need more practice, do question 8.

8. Look at the first four triangular numbers shown in question 1 of On Your Own on page 16 of your textbook.

a. Draw the next (fifth) triangular number.
b. Explain how the pattern grows.
c. Copy and complete the T-table shown on page 16.
d. How is the design number related to the total number of dots?

Check your answers on pages 124 and 125 in the Appendix.

9. It takes six police officers on motorcycles to escort one sports team in a parade. For each additional sports team that joins the parade, four more police officers are needed.

   a. Construct and label a table that can be used to see the number of police officers needed to escort from 1 to 10 sports teams.

   b. Make a rule that tells how the number of police officers needed is related to the number of sports teams.

10. How many different ways could you make change for 50¢ if the only coins available are quarters, dimes, and nickels? Make and complete a table that will help you find all possibilities.

11. When shopping for a new vehicle, the Browns looked at three different models: a sedan, a station wagon, and a van. Each vehicle comes in four different colours: red, blue, green, and black. Use the clues to find the colour of each of the models the Browns saw.

   - Each of the models was a different colour.
   - The colour of the station wagon and the colour of the van begin with the same letter.
   - The sedan was not green.
   - There were no vans available in black.

   Make a table to help you match the models and colours.

12. At the library, seven books about each of the following topics were checked out by some children: animals, space, and mystery. Two children chose only mystery books. Two children chose both an animal book and a mystery book. Three children chose books on all three topics. One child chose a space book and an animal book.

a. Draw and label a Venn diagram and decide which numbers must be in each area.

b. How many children chose only animal books?

c. How many children chose both mystery and space books?

d. How many children chose only space books?

e. How many children didn't choose animal books?

f. What is the total number of children that checked out books?

If you need help with questions 9 to 12, look back at Lesson 3, where you learned about using patterns and charts to solve problems. If you feel you need more practice, do questions 13 to 16.

13. **a.** Do question 2 from Problem Bank on page 37 of your textbook.
b. Do question 3 from Problem Bank on page 37 of your textbook.

14. Pencils cost 15¢ each and erasers cost 20¢ each. If Penny had 75¢ to spend, and needed at least one of each, which items could she buy?

15. Three sisters, Jenny, Joan, and Jill, each saved a different amount of money: $4, $7, and $10. Use the following clues to find how much each girl saved.

- Jenny saved more than Jill.
- Joan saved an even number of dollars.
- Jenny saved less than Joan.

a. Make a table and complete it to help you match each girl with her savings.

b. Tell how much each girl saved.

16. Brad had a bag of granola bars. Nine bars had chocolate chips, six bars had raisins, and seven bars had nuts. Four bars had only chocolate chips; one bar had chocolate chips, raisins, and nuts; two bars had only nuts and chocolate chips; and three bars had only raisins and nuts.

a. Draw and label a Venn diagram and decide which numbers must be in each area.

b. How many bars had only nuts?

c. How many bars had only raisins and chocolate chips?

d. How many bars had only raisins?

e. How many bars didn't have raisins?

f. What is the total number of granola bars in the bag?

Check your answers on pages 126 and 130 in the Appendix.

17. Turn to page 88 of your textbook. Do question 3 of Problem Bank. Explain your reasoning for choosing each number you used to complete the puzzle.

If you need help with question 17, look back at Lesson 4, where you learned about factors and multiples. If you feel you need more practice, solve questions 18, 19, and 20.

18. Explain how you can find the first ten multiples of 6. List them.

19. Is 66 a multiple of 4? Explain your answer.

20. Look at the following numbers: 21, 28, 35, 42, …

a. These are all multiples of what number?
b. If the pattern continues, what are the next three numbers?

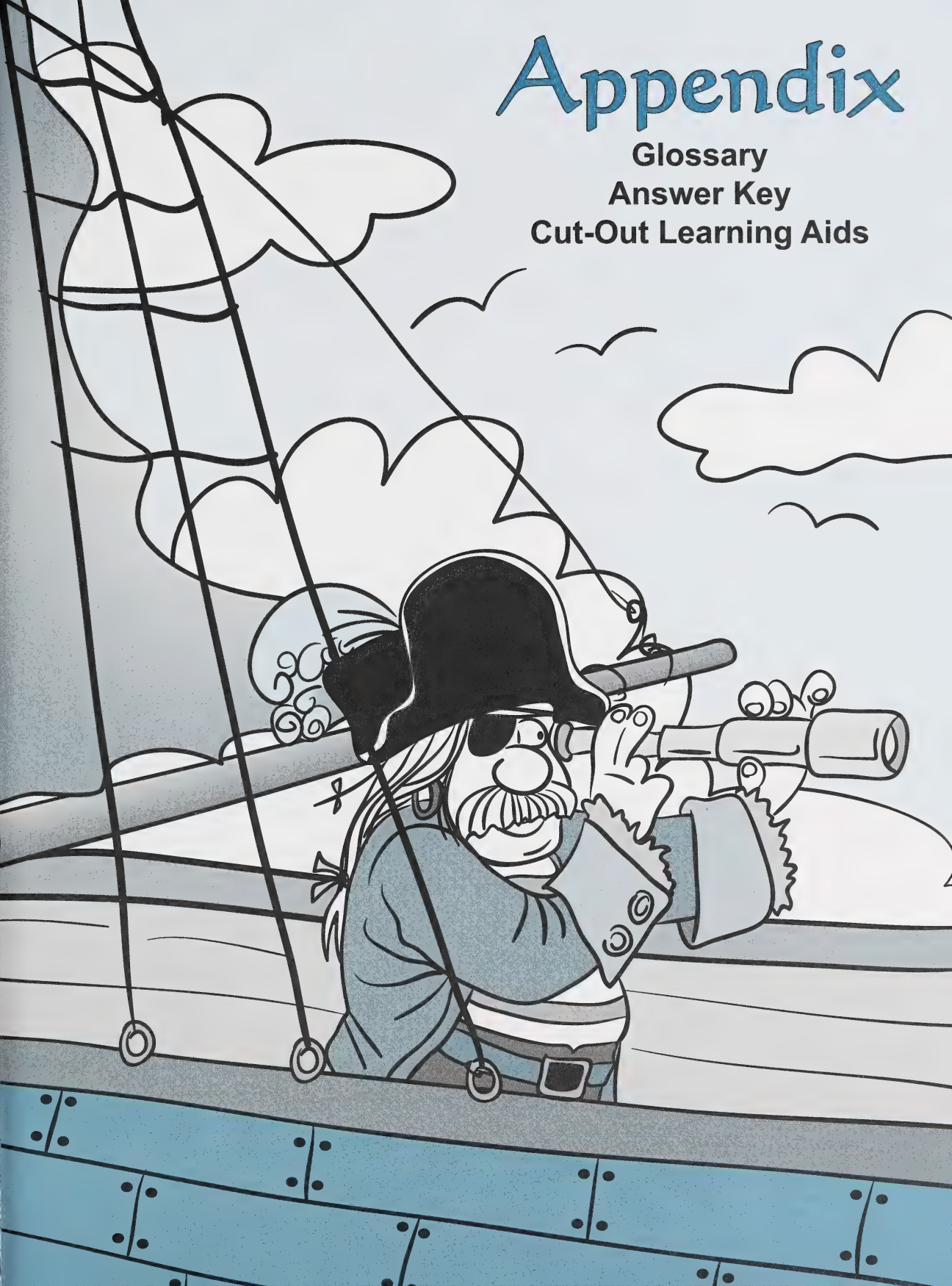
Check your answers on page 131 in the Appendix.

21. Turn to page 88 of your textbook. Do questions 1 and 2 of Problem Bank. You may use tiles to help you decide if the numbers are prime or composite, and to help you find factors of numbers.

If you need help with question 21, look back at Lesson 5, where you learned about prime and composite numbers. If you feel you need more practice, do question 22.

22. Turn to page 67 in your textbook. Do questions 1 to 3 of Practise Your Skills. You may find it useful to use tiles.

Check your answers on pages 132 and 133 in the Appendix.



Appendix

Glossary
Answer Key
Cut-Out Learning Aids

Glossary

Carroll diagram: a diagram that uses columns and rows to show how two different kinds of information are related

composite number: a number that has more than two different factors

prime number: a number that has exactly two different factors (the number itself and 1)

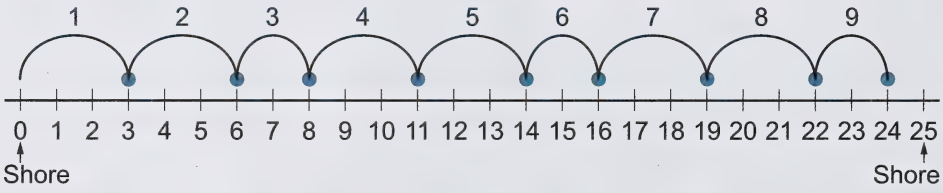
Venn diagram: a diagram that uses circles to show how two or three groups of things are related

Answer Key

Lesson 1: Describing Number Patterns

Activity 1

1. a.



He landed on stones 3, 6, 8, 11, 14, 16, 19, 22, and 24.

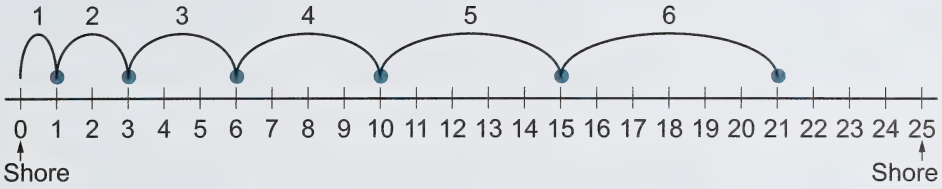
b. You may have described the pattern in any of the following ways:

- He took hops of 3 stones, 3 stones, 2 stones; then 3 stones, 3 stones, 2 stones; and then 3 stones, 3 stones, 2 stones.
- He took three sets of hops. Each set was made up of a 3-stone hop, a 3-stone hop, and a 2-stone hop.
- His hopping pattern was 3, 3, 2; 3, 3, 2; 3, 3, 2.

c. Meeker's last hop to shore (his 10th hop) would be equal to 1 stone length. He would have to jump from tick number 24 to tick number 25, and $25 - 24 = 1$.

d. It would have taken Meeker 10 hops. That is, it would have taken the 9 hops shown plus 1 more hop to shore.

2. a.



Meeker used stones 1, 3, 6, 10, 15, and 21 in his first six hops.

b. Meeker's last hop to shore (his 7th jump) would be 4 stones in length. He would have to jump from tick number 21 to tick number 25, and $25 - 21 = 4$.

c. It would take 7 hops to cross the stream. That is, it would take the 6 hops shown on the number line plus 1 more hop to shore.

d. Except for the last hop, the hop length is equal to the hop number. That is, hop 1 is 1 stone length, hop 2 is 2 stones length, hop 3 is 3 stones length, and so on.

e. The total distance covered with 10 hops could be

$$1+2+3+4+5+6+7+8+9+10 = 55 \text{ stones}$$

However, the endpoint must be the shore, not another stone. Therefore, there can only be 54 stepping stones.

3. Textbook, page 38, question 1

1. a. 1, 3, 5, 7, 9, 11, 13

+2 +2 +2

b. 6, 13, 20, 27, 34, 41, 48

+7 +7 +7

c. 33, 30, 27, 24, 21, 18, 15

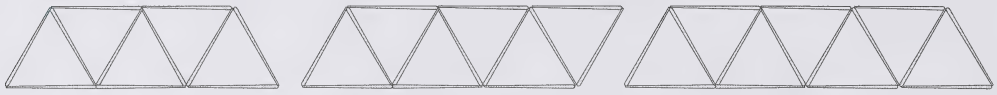
−3 −3 −3

d. 3, 9, 27, 81, 243, 729, 2187

×3 ×3 ×3

Activity 2

1. a. The next three designs are shown.



b. Eleven toothpicks are needed to build the design with five triangles.

c.

Number of Triangles	1	2	3	4	5	6	7	8	9	10
Number of Toothpicks	3	5	7	9	11	13	15	17	19	21

d. To build the first design, which has one triangle, you need three toothpicks. For each additional triangle, you need two more toothpicks.

e. The number of toothpicks needed to build each design is twice the number of triangles plus one.

f. For 75 triangles, you would need $(2 \times 75) + 1 = 150 + 1 = 151$ toothpicks.

2. a.

Hours Worked	1	2	3	4	5	6	7	8	9	10
Pay			$12	$16	$20	$24				

b. For each additional hour Kevin works, his pay increases by $4.

c. He would earn $4 less for each hour less that he worked.

d. Multiply the number of hours Kevin works by $4. He earns $4 each hour he works.

e.

Hours Worked	1	2	3	4	5	6	7	8	9	10
Pay	$4	$8	$12	$16	$20	$24	$28	$32	$36	$40

3. Tables organize information. This makes it easier to compare numbers to see if there is a pattern or a rule that relates the numbers in the table.

Activity 3

1. Divide by 9.

2. **Practice and Homework Book, page 10, questions 1 to 6**

   1. Rule: Divide by 6.

   2. Rule: Multiply by 9.

   3. Rule: Add 5; then divide by 2.

   4. Rule: Multiply by 3; then add 2.

   5.

Input	Output
2	6
8	30
9	34

   6.

Input	Output
3	8
9	10
18	13

Challenge Activity

1. Textbook, pages 26 and 27, question 1

   Note: The top spoke of the Japanese Number Circle should read 47, **48**, 49, 50, 51.

   The numbers 1 through 51 are used. The numbers 2 through 26 form five spokes of the wheel, counting out from the centre. The numbers 27 through 51 form the other five spokes, counting from the outside to the centre.

   There are many sum patterns. Two examples are shown.

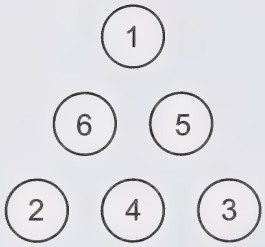
   - The sum of all the numbers in any two spokes that are opposite each other is 265.

     $2+3+4+5+6+47+48+49+50+51=265$

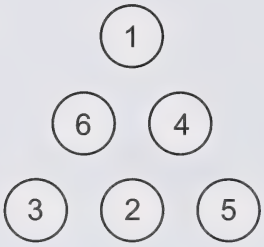
   - The sum of all the numbers that form any one of the circles is 265.

     $2+31+36+41+46+51+22+17+12+7=265$

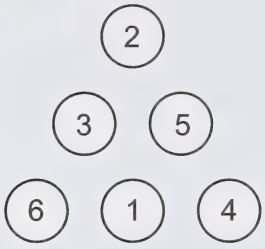
2. a.



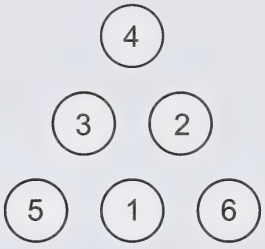
b.



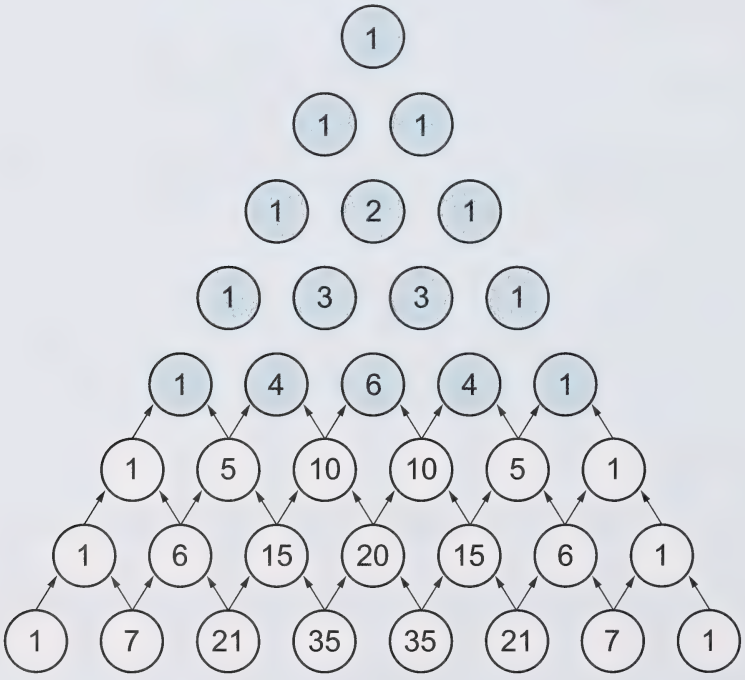
c.



d.



3. a. The numbers in the last three rows can be found by adding the numbers joined by the arrows in the row above.



b. The sums of the horizontal rows are 1, 2, 4, 8, 16, 32, 64, and 128. The sum of each row is 2 times the sum of the preceding row.

Lesson 2: 2-D and 3-D Patterns

Activity 1

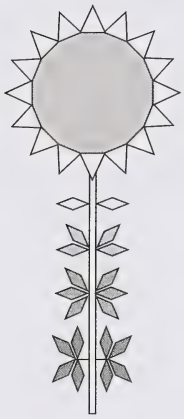
1. The smallest flower has 4 triangles (green petals). Each time the flower grows to the next larger size, it grows 4 more green petals.

Design Number	Number of Triangles	
1	4	
2	8	+4
3	12	+4
4	16	+4
5	20	+4
6	24	+4
7	28	+4
8	32	+4

2. The smallest size of flower has 2 parallelograms (brown leaves). As the flowers grow in size, they get new leaves. The second flower gets 4 new leaves, the next flower gets 6 new leaves, the next flower gets 8 new leaves, and so on. The number of new leaves a flower gets each time is the next even number.

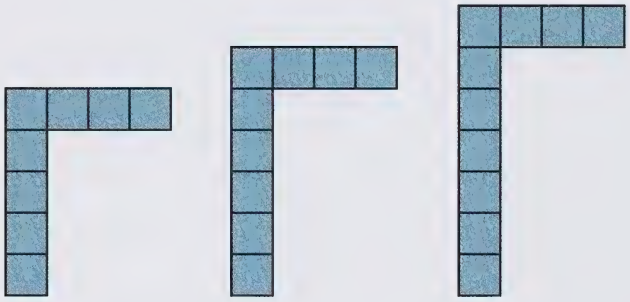
Design Number	Number of Parallelograms	
1	2	
2	6	+4
3	12	+6
4	20	+8
5	30	+10
6	42	+12
7	56	+14
8	72	+16

3.



Activity 2

1. a.

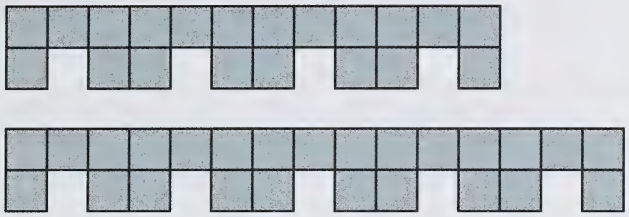


b. The lowest diving platform is 2 tiles high and 4 tiles long, and it has 5 tiles in all. The diving platforms do not increase in length, but the height increases by 1 tile each time, so the total number of tiles increases by 1 each time.

Height of Diving Platform	Number of Tiles in All
2	5
3	6
4	7
5	8
6	9
7	10
8	11
9	12
10	13

+1 +1 +1 +1 +1 +1 +1 +1

2. a.



b. The smallest gate has one opening and has 5 tiles in all. Each next larger gate has one more opening and uses 5 more tiles than the gate before it.

Number of Gates	Number of Tiles in All
1	5
2	10 (+5)
3	15 (+5)
4	20 (+5)
5	25 (+5)
6	30 (+5)
7	35

3. Practice and Homework Book, page 2, questions 1 and 2

1. Rule: The smallest design has 4 tiles in all. Each next larger design has 1 more tile.

Height of Tower	Number of Tiles in All
2	4
3	5
4	6
5	7
6	8
7	9
8	10
9	11

2. Rule: The smallest design has 4 tiles in all. Each next larger design has 4 more tiles.

Number of Windows	Number of Tiles in All
1	4
2	8
3	12
4	16
5	20
6	24
7	28
8	32
9	36

4. a. The staircase grows by one step each time. One cube is used for the first staircase, 2 more cubes are added to make the second staircase, 3 more cubes are added to make the third staircase, and so on.

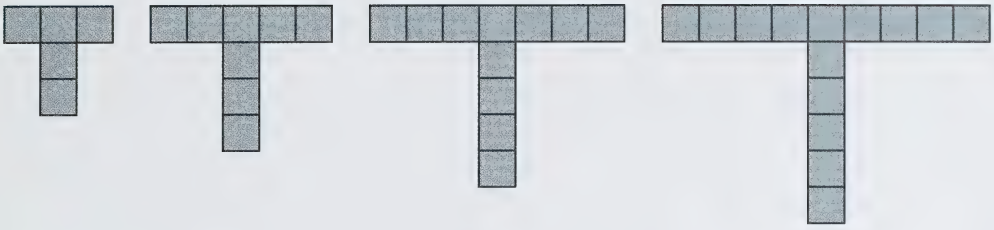
b.

Staircase Number	Number of Cubes Needed
1	1
2	3 (+2)
3	6 (+3)
4	10 (+4)
5	15 (+5)
6	21 (+6)
7	28 (+7)

c. The number of cubes needed to build staircase number 10 is 55. Following the pattern, you would use $1+2+3+4+5+6+7+8+9+10=55$ cubes.

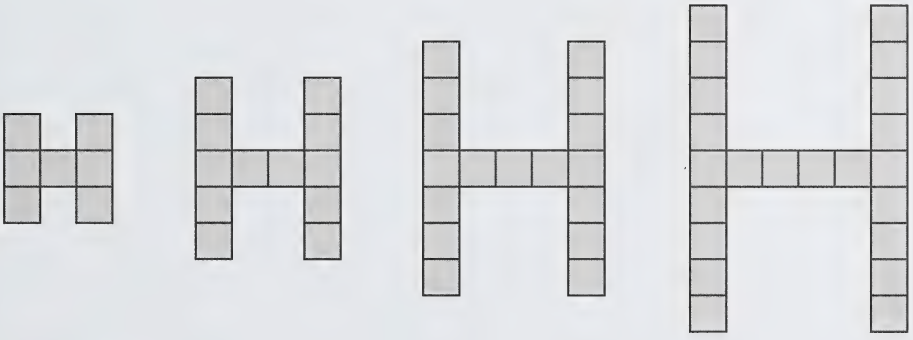
Activity 3

1. a.



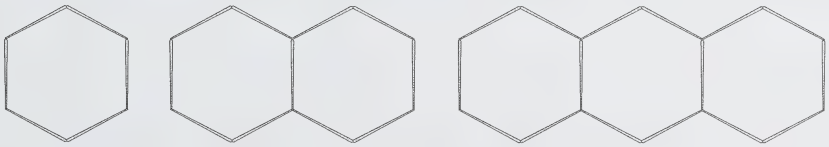
b. The T-designs grow by adding an extra tile to each of the three ends, so each T-design will be 2 tiles wider and 1 tile higher than the T-design before it.

2. a.



b. The H-designs grow by adding a tile to both ends of each of its arms, and one more tile in the middle so each H-design will be 2 tiles higher and 1 tile wider than the H-design before it.

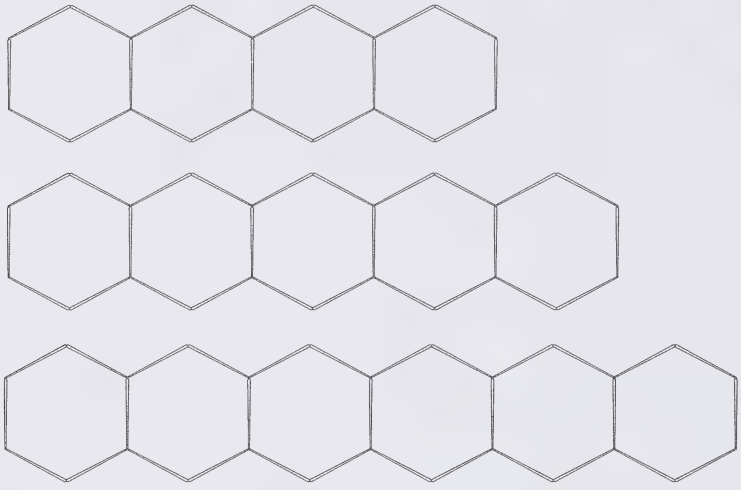
3. a. The first three designs in the sequence may look like these.



b.

Design Number	1	2	3	4	5	6
Number of Toothpicks Used	6	11	16	21	26	31

The next three shapes look like this:



The next three designs in the pattern use 21, 26, and 31 toothpicks.

c. Each shape uses 5 more toothpicks than the shape before it. To find the number of toothpicks used for the shape next number, add 5 more to the number of toothpicks used in the previous shape.

d. The number of toothpicks is 5 times the design number, plus 1.

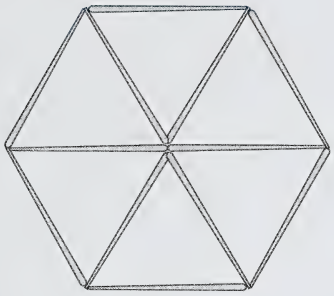
e. Multiply 25 by 5 and add 1.

$$5 \times 25 = 125;\ 125 + 1 = 126$$

Design number 25 in the pattern uses 126 toothpicks.

Challenge Activity

1.



2. **Textbook, page 15, questions 1 and 2**

   1. You can extend the T-table to answer the problem.

Number of Boxes in Bottom Row	Total Number of Boxes	
1	1	
2	3	+2
3	6	+3
4	10	+4
5	15	+5
6	21	+6
7	28	+7
8	36	+8
9	45	+9

   There should be 9 boxes in the bottom row of a pyramid with 45 boxes.

2. You can extend the T-table to answer the problem.

Number of Tiles in Middle Row	Total Number of Tiles
1	1 ← 1×1
2	4 ← 2×2
3	9 ← 3×3
4	16 ← 4×4
5	25 ← 5×5
6	36 ← 6×6
7	49 ← 7×7

There should be 7 tiles in the middle row to use the greatest number of tiles. (50 – 49 = 1, so 1 tile will be left over)

Lesson 3: Using Patterns to Solve Problems

Activity 1

1. Method A

Day of the Month	1	2	3	4	5	6	7	8	9	10
Doubloons Received for the Day	10	10	10	10	10	10	10	10	10	10
Total Doubloons	10	20	30	40	50	60	70	80	90	100

Method B

Day of the Month	1	2	3	4	5	6	7	8	9	10
Doubloons Received for the Day	1	2	3	4	5	6	7	8	9	10
Total Doubloons	1	3	6	10	15	21	28	36	45	55

2. a. The total number of doubloons increases by 10 doubloons each day.

b. On the first day, the total is 1 doubloon. The next day, the total increases by 2 doubloons. The next day, the total increases by 3 doubloons. The amount by which the total increases each day is 1 more doubloon than it increased by the day before.

3. a. Multiply the day of the month by 10.

b. Multiply the number of doubloons received that day by the number of doubloons that will be received the next day and divide the answer by 2. The number of doubloons paid each day is the same as the day of the month.

4. a. There are 30 days in September. Multiply 30 by 10.

$$30 \times 10 = 300$$

The total pay received will be 300 doubloons.

b. There are 30 days in September, so the number of doubloons paid that day would be 30. Multiply 30 by 31 and divide the answer by 2. The number of doubloons paid the next day would be 31.

$$30 \times 31 = 930 \text{ and } 930 \div 2 = 465$$

The total pay received would be 465 doubloons.

5. The cook should choose Method B. The pay would be greater by $465 - 300 = 165$ doubloons.

6. Textbook, page 31, On Your Own, questions 1 to 4

1.

Day Worked	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Dollars for the Day	1	3	5	7	9	11	13	15	17	19	21	23	25	27	29
Total Dollars	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

You should choose the plan where you earn $2 more each day. You will make $225 for 15 days of work. That is $125 more than if you had chosen to be paid $100 for 15 days of work.

2.

Stop Number	1	2	3	4	5	6	7	8
People Getting On	1	2	3	4	5	6	7	8
Total People	1	3	6	10	15	21	28	36

The bus that makes 8 stops would have a total of 36 people, which would make it less crowded than the bus with 45 passengers.

3.

Year Number	1	2	3	4	5
Trees Planted	2	3	3	3	3
Total Trees	2	5	8	11	14

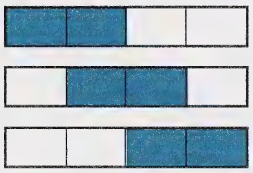
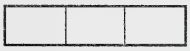
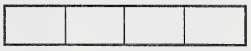
The pattern of adding 3 trees each year after the first year yields a total of 14 trees at the end of 5 years. Planting 20 trees all at once would give you more trees.

4. A sample answer is given.

I made tables to record the information given for the different situations. The tables helped me to find patterns, and the patterns showed me how the numbers were related.

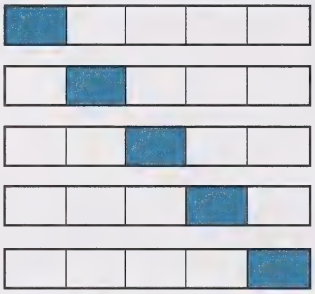
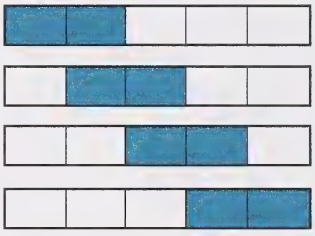
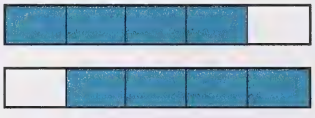
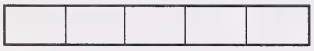
Activity 2

1.

Size of Rectangle	Rectangles of That Size	Number of That Size
		4
		3
		2
		1

Total number of rectangles = 10

2.

Size of Rectangle	Rectangles of That Size	Number of That Size
		5
		4
		3
		2
		1

Total number of rectangles = 15

3. Each time the number of panes in the window increases by one, the total number of rectangles will increase by the number of panes in that window.

4. The total number of rectangles that can be seen in any window is the sum of all the counting numbers from 1 up to and including the number of panes in that window.

5. **a.** There are 7 panes in the window, so the total number of rectangles is $1+2+3+4+5+6+7=28$ rectangles.

   b. There are 10 panes in the window, so the total number of rectangles is $1+2+3+4+5+6+7+8+9+10=55$ rectangles.

6.

Number of Chairs Made	Number of Legs Used for Chairs	Number of Legs Left Over	Number of Stools That Can Be Made with Leftover Legs	Number of Legs Left Over	Are all 31 legs used?
1	$1 \times 4 = 4$	$31 - 4 = 27$	$27 \div 3 = 9$	0	yes
2	$2 \times 4 = 8$	$31 - 8 = 23$	$23 \div 3 = 7$ R2	2	no
3	$3 \times 4 = 12$	$31 - 12 = 19$	$19 \div 3 = 6$ R1	1	no
4	$4 \times 4 = 16$	$31 - 16 = 15$	$15 \div 3 = 5$	0	yes
5	$5 \times 4 = 20$	$31 - 20 = 11$	$11 \div 3 = 3$ R2	2	no
6	$6 \times 4 = 24$	$31 - 24 = 7$	$7 \div 3 = 2$ R1	1	no
7	$7 \times 4 = 28$	$31 - 28 = 3$	$3 \div 3 = 1$	0	yes

7. No, he could not make more than 7 chairs. Chairs have 4 legs each, so you would need 32 legs to make 8 chairs.

8. Claire's grandpa can make 1 chair and 9 stools, 4 chairs and 5 stools, or 7 chairs and 1 stool.

9.

Number of Stools Made	Number of Legs Used for Stools	Number of Legs Left Over	Number of Chairs That Can Be Made with Leftover Legs	Number of Legs Left Over	Are all 31 legs used?
1	$1 \times 3 = 3$	$31 - 3 = 28$	$28 \div 4 = 7$	0	yes
2	$2 \times 3 = 6$	$31 - 6 = 25$	$25 \div 4 = 6$ R1	1	no
3	$3 \times 3 = 9$	$31 - 9 = 22$	$22 \div 4 = 5$ R2	2	no
4	$4 \times 3 = 12$	$31 - 12 = 19$	$19 \div 4 = 4$ R3	3	no
5	$5 \times 3 = 15$	$31 - 15 = 16$	$16 \div 4 = 4$	0	yes
6	$6 \times 3 = 18$	$31 - 18 = 13$	$13 \div 4 = 3$ R1	1	no
7	$7 \times 3 = 21$	$31 - 21 = 10$	$10 \div 4 = 2$ R2	2	no
8	$8 \times 3 = 24$	$31 - 24 = 7$	$7 \div 4 = 1$ R3	3	no
9	$9 \times 3 = 27$	$31 - 27 = 4$	$4 \div 4 = 1$	0	yes
10	$10 \times 3 = 30$	$31 - 30 = 1$	—	1	no

Activity 3

1. Mark had a vanilla cone, Samuel had a chocolate cone, Anna had a strawberry cone, and Celine had a mint cone. Your strategy should be similar to the following:

- No child prefers a flavour with the same number of letters as his or her name. Mark and Anna have the same number of letters as mint, so put **no** in the mint row under each of their names.

- Anna is allergic to chocolate, so she cannot eat chocolate ice cream or chocolate syrup on vanilla ice cream. Put **no** in the chocolate and vanilla rows under her name. This leaves only one choice for Anna—she must prefer strawberry. Put **yes** in the strawberry row under her name. Since each person prefers a different flavour of ice cream, put **no** in the strawberry row under each of the other children's names.

- The girl who prefers mint ice cream likes her ice cream in sugar cones. Since mint must be a girl's treat and Anna has been eliminated, the mint ice cream must be Celine's. Put **yes** in the mint row under her name. Put **no** in the mint row under Samuel's name.

- Samuel thinks vanilla is boring, and mint and strawberry have been eliminated, so he must prefer chocolate. Put **yes** in the chocolate row under Samuel's name. Put **no** in the chocolate row under each of the other children's names. This leaves vanilla for Mark. Put **yes** in the vanilla row under Mark's name and **no** under each of the other children's names.

	Mark	Samuel	Anna	Celine
Chocolate	no	yes	no	no
Mint	no	no	no	yes
Strawberry	no	no	yes	no
Vanilla	yes	no	no	no

2. a. Six plates were cracked and chipped.
b. Four plates were only cracked.
c. Five plates were only chipped.
d. Twenty-five plates were not cracked or chipped.
e. $6+4+5+25=40$

Reilly bought 40 plates in total.

3. a. All the areas inside the dance circle have a sum of 10, so 10 students chose dance. $1+2+3+4=10$

b. All the areas inside the chess circle have a sum of 8, so 8 students chose chess. $2+2+1+3=8$

c. All the areas inside the drama circle have a sum of 14, so 14 students chose drama. $3+1+3+7=14$

d. All the areas outside the dance circle didn't choose dance, so 12 didn't choose dance. $2+3+7=12$

e. Add the numbers in all the areas. A total of 22 people signed up.
$2+2+3+1+3+4+7=22$

Challenge Activity

1. One way to solve the problem is to assume that one of the characters is telling the truth. Then test the statement for contradictions. You could use the following reasoning:

   Step 1: Assume that Bruce Bull is telling the truth. This means that the others are telling falsehoods.

Bruce Bull: "Bobby Bear spilled the milk."	T
Digger Dog: "Bobby Bear is lying."	F
Bobby Bear: "Digger Dog spilled the milk."	F
Randy Rooster: "I didn't spill the milk."	F

   Test the statements for contradiction.

   If you assume Bruce Bull is telling the truth, there is a contradiction in the second statement. If Digger Dog is telling a falsehood, Bobby Bear must be telling the truth. This is not possible, as there is only one truth teller.

   Step 2: Assume Digger Dog is telling the truth. This means that the others are telling falsehoods.

Bruce Bull: "Bobby Bear spilled the milk."	F
Digger Dog: "Bobby Bear is lying."	T
Bobby Bear: "Digger Dog spilled the milk."	F
Randy Rooster: "I didn't spill the milk."	F

   Test the statements for contradiction.

   If Bruce Bull is telling a falsehood, Bobby Bear is innocent.
   If Digger Dog is telling the truth, Bobby Bear is lying.
   If Bobby Bear is telling a falsehood, Digger Dog is innocent.
   If Randy Roster is telling a falsehood, he spilled the milk himself.

   There are no contradictions.

   Digger Dog is telling the truth. Randy Roster is guilty.

2. One way to solve this problem is to assume that one character is telling the truth. Then test the statements for contradictions.

You could use this reasoning:

Step 1: Assume that Betty Beaver is telling the truth. This means that the others are telling falsehoods.

Betty Beaver: Cheryl Chipmunk did it.	T
Cheryl Chipmunk: I didn't do it.	F
Sandy Squirrel: I didn't do it.	F

Test the statements for contradictions.

If Betty Beaver is telling the truth, Cheryl Chipmunk is guilty.
If Cheryl Chipmunk is telling a falsehood, Cheryl Chipmunk is guilty.
If Sandy Squirrel is telling a falsehood, Sandy Squirrel is guilty.

This is not possible. Only one was the robber. You must eliminate Betty Beaver as the truth teller.

Step 2: Assume Cheryl Chipmunk is telling the truth. This means that the others are telling falsehoods.

Betty Beaver: Cheryl Chipmunk did it.	F
Cheryl Chipmunk: I didn't do it.	T
Sandy Squirrel: I didn't do it.	F

Test the statements for contradictions.

If Betty Beaver is telling a falsehood, Cheryl Chipmunk is innocent.
If Cheryl Chipmunk is telling the truth, Cheryl Chipmunk is innocent.
If Sandy Squirrel is telling a falsehood, Sandy Squirrel is guilty.

There are no contradictions.

Cheryl Chipmunk is telling the truth. Sandy Squirrel is guilty.

Lesson 4: Multiples and Factors

Activity 1

1. a.

1	2	3	(4)	5	6	7	(8)	9	10
11	(12)	13	14	15	(16)	17	18	19	(20)
21	22	23	(24)	25	26	27	(28)	29	30
31	(32)	33	34	35	(36)	37	38	39	(40)
41	42	43	(44)	45	46	47	(48)	49	50
51	(52)	53	54	55	(56)	57	58	59	(60)
61	62	63	(64)	65	66	67	(68)	69	70
71	(72)	73	74	75	(76)	77	78	79	(80)
81	82	83	(84)	85	86	87	(88)	89	90
91	(92)	93	94	95	(96)	97	98	99	(100)

b. The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, and 100.

c. Each sailor will get cod-liver oil 25 times.

2. a. Begin with 4 and keep adding 4 each time to make the next number.

4, 8, 12, 16, 20 24, …

+4 +4 +4 +4 +4

b. Begin with 100 (the last day that medicine was given) and keep adding 4 each time to find the next 5 numbers.

100, 104, 108, 112, 116, 120

+4 +4 +4 +4 +4

Cod-liver oil will be given on days 100, 104, 108, 112, 116, and 120.

3. a. Multiply 4 by each of the counting numbers (1, 2, 3, and so on).

4, 8, 12, 16, 20, 24, …

↑ ↑ ↑ ↑ ↑ ↑

1×4 2×4 3×4 4×4 5×4 6×4

b. Medicine will be given on the 100th day. Since $100 = 25 \times 4$, multiply 4 by the next 5 counting numbers.

$$26 \times 4 = 104,\ 27 \times 4 = 108,\ 28 \times 4 = 112,\ 29 \times 4 = 116,\ 30 \times 4 = 120$$

Medicine will be given on days 104, 108, 112, 116, and 120.

4. a. Multiples of 4 are made by multiplying 4 by the counting numbers. This means that if a number is a multiple of 4, it can be divided evenly by 4. A multiple of 4 must have 4 as a factor.

b. $112 \div 4 = 28,\ 146 \div 4 = 36$ R2, $160 \div 4 = 40$

Medicine will be given on days 112 and 160, but not on day 146.

5. a. Callers 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, and 96 will win movie passes.

b. Callers 102, 108, 114, 120, and 126 will be the next five winners. Sample strategies are given:

- Begin with 96 (the last winner) and keep adding 6.

$$96,\ \underset{+6}{102},\ \underset{+6}{108},\ \underset{+6}{114},\ \underset{+6}{120},\ \underset{+6}{126}$$

- Caller 96 was the last winner. Since $96 = 16 \times 6$, multiply 6 by the next five counting numbers.

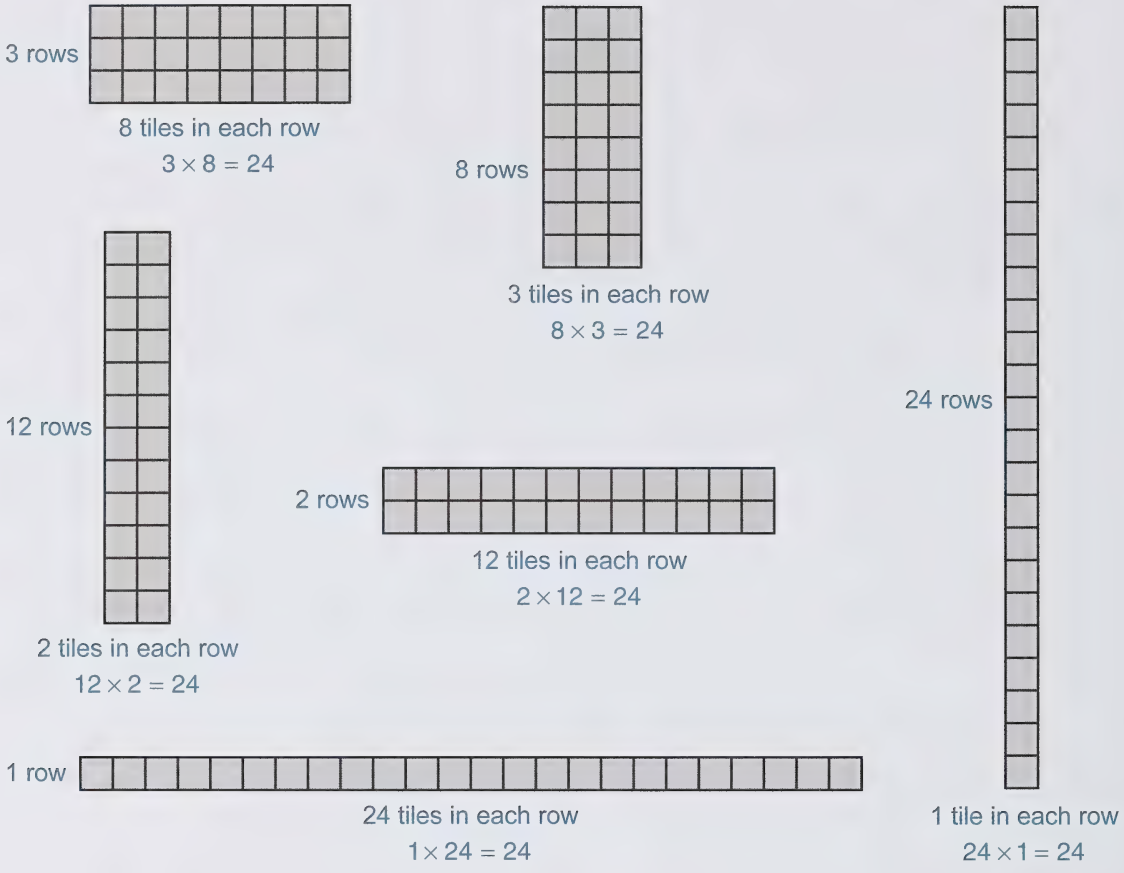
$$17 \times 6 = 104,\ 18 \times 6 = 108,\ 19 \times 6 = 114,\ 20 \times 6 = 120,\ 21 \times 4 = 126$$

c. Callers 138 and 198 will win movie passes, but caller 152 won't.

$$138 \div 6 = 23,\ 152 \div 6 = 25 \text{ R2},\ 198 \div 6 = 33$$

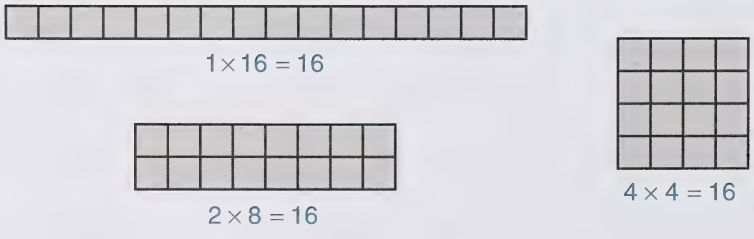
Activity 2

1.



2. To find all the factors of any given number, use a set of that number of tiles. Make all the possible arrays that can be made with that set of tiles. For each array you make, the number of rows is one factor and the number of tiles in each row is the other factor.

3. a.



b. The factors of 16 are 1, 2, 4, 8, and 16.

4. No, it is not possible to make the same number of arrays for every set.

5. The least number of arrays for every set is 1. You can always make an array with all the tiles in a single row.

6. Multiply the number of rows by the number of tiles in each row to find the number of tiles in the set.

Activity 3

1. If you know one of the factors in a factor pair, divide the number by the factor you know and the answer will be the other factor.

2. The number 2 is a factor of any even number. Any number that ends with 0, 2, 4, 6, or 8 is even and will have 2 as a factor.

3. **a.**

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
1	30 ÷ 1 = 30	yes	30
2	30 ÷ 2 = 15	yes	15
3	30 ÷ 3 = 10	yes	10
4	30 ÷ 4 = 7 R2	no	—
5	30 ÷ 5 = 6	yes	6
6	30 ÷ 6 = 5	yes	5

The factors have started to repeat, so you have found them all.

b. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

4. Textbook, page 67, On Your Own, questions 1 to 4

1.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
1	36 ÷ 1 = 36	yes	36
2	36 ÷ 2 = 18	yes	18
3	36 ÷ 3 = 12	yes	12
4	36 ÷ 4 = 9	yes	9
5	36 ÷ 5 = 7 R1	no	—
6	36 ÷ 6 = 6	yes	6

The factors have started to repeat, so you have found them all.

It is possible to make 2 groups of 18, 3 groups of 12, 4 groups of 9, 6 groups of 6, 9 groups of 4, 12 groups of 3, and 18 groups of 2. Note that 1 and 36 are also factors of 36, but 1 group of 36 means that the whole class is working together or 36 groups of 1 means that each student is working alone.

2.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
1	42 ÷ 1 = 42	yes	42
2	42 ÷ 2 = 21	yes	21
3	42 ÷ 3 = 14	yes	14
4	42 ÷ 4 = 10 R2	no	—
5	42 ÷ 5 = 8 R2	no	—
6	42 ÷ 6 = 7	yes	7
7	42 ÷ 7 = 6	yes	6

The factors have started to repeat, so you have found them all.

It is possible to make 2 groups of 21, 3 groups of 14, 6 groups of 7, 7 groups of 6, 14 groups of 3, and 21 groups of 2.

3. Any number that is divisible by 8 will also be divisible by 2 and 4 because 2 and 4 are factors of 8. However, 6 is not a factor of 8. This means you need to find a number that is a multiple of 8 and also of 6.

   The multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, and so on. From this list, 6 is a factor of 24, 48, 72, and so on. These are the multiples of 24. The number of students in the Camera Club might be 24 or any multiple of 24.

4. A sample answer is given.

   You may use factors to decide if you can make equal sets of things from a larger set. Two examples are given.

   - Can I share 23 cookies equally among 5 people?
   - Can I make teams of 5 with 30 people?

Challenge Activity

1. There are several solutions. Sample answers are given.

   The first sack contains a multiple of 2, but not a multiple of 3, so write the multiples of 2 and cross out the multiples of 3.

   2 4 ~~6~~ 8 10 ~~12~~ 14 16 ~~18~~ 20

   The second sack contains a multiple of 4, but not a multiple of 5, so write the multiples of 4 and cross out the multiples of 5.

   4 8 12 16 ~~20~~ 24 28 32

   The third sack contains a multiple of 3, but not a multiple of 4, so write the multiples of 3 and cross out the multiples of 4.

   3 6 9 ~~12~~ 15 18 21 ~~24~~ 27 30

   Now, choose one number from each sack so that the sum of those numbers is a multiple of 5.

   - You cannot make a sum of 5.
   - You cannot make a sum of 10.
   - Some of the sums that can be made are shown in the following table.

Number of Coins in First Sack	Number of Coins in Second Sack	Number of Coins in Third Sack	Total Number of Coins in all Three Sacks
2	4	9	15
4	8	3	15
8	4	3	15
2	12	6	20
10	4	6	20
2	12	6	20
14	8	3	25

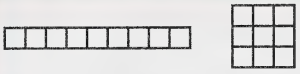
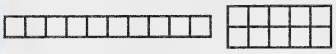
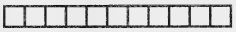
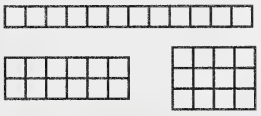
2. Practice and Homework Book, page 44, questions 1 to 4

	Total Length	Big Jump	Little Jump
1.	22 units	4 units	2 units
2.	19 units	5 units	2 units
3.	32 units	8 units	4 units
4.	18 units	5 units	2 units

Lesson 5: Composite and Prime Numbers

Activity 1

1. a.

Number of Sailors Available to Go Exploring	Possible Array(s) of Sailors That Can Be Made	Can that size of team go ashore? If so, what smaller groups can be made?	Factor Pairs Shown by the Arrays
9		Yes. 3 groups of 3.	1×9, 3×3
10		Yes. 2 groups of 5 or 5 groups of 2.	1×10, 2×5
11		No	1×11
12		Yes. 2 groups of 6, 3 groups of 4, 6 groups of 2, or 4 groups of 3.	1×12, 2×6, 3×4

b. No, the rule doesn't always work. A team of 9 sailors can split into 3 smaller groups of 3. Nine is not an even number.

2. a. You could make only single rows with 2, 3, 5, 7, and 11 sailors.
b. These numbers have exactly two factors (the number itself and 1).

3. a. Both a single row and another array can be made for 4, 6, 8, 9, 10, and 12 sailors.

b. These numbers have more than two factors.

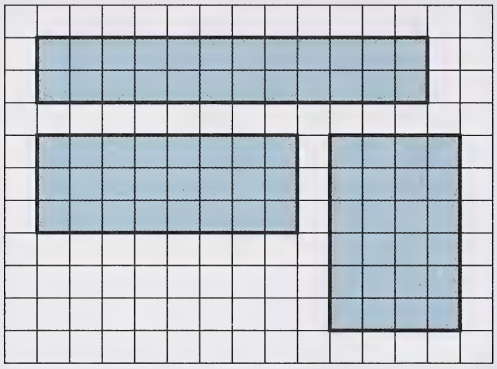
4. The number 1 is neither prime nor composite because it has only one factor. A prime number must have exactly two different factors. A composite number must have more than two different factors.

5. A prime number has exactly two different factors, itself and one. The only factors of 2 are 2 and 1, so it is prime. Even numbers are numbers that have 2 as a factor, so every even number besides 2 has at least 3 factors: itself, 1, and 2. Therefore, even numbers greater than 2 must be composite numbers.

Activity 2

1. You can make the three rectangles shown.

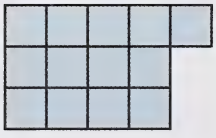
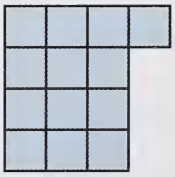
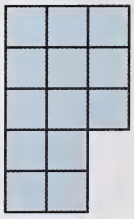
- 2×12
- 3×8
- 6×4

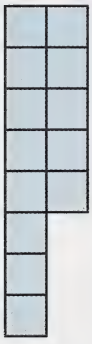


Note that if you gave the grid a quarter turn, the rectangles would be

- 12×2
- 8×3
- 4×6

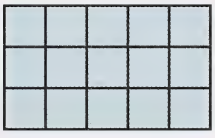
2. a.

Number of Rows Attempted	2	3	4	5
Picture				

Number of Rows Attempted	6	7	8	9	10	11	12
Picture							

The number 13 is a prime number. It is impossible to make a rectangle with at least 2 squares in each row.

b.

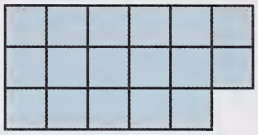
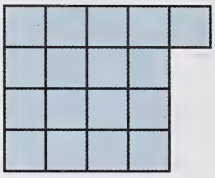
Number of Rows Attempted	2	3
Picture		

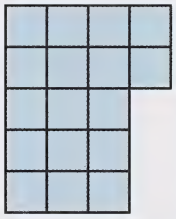
The number 15 is a composite number because it is possible to make a 3×5 rectangle.

3. Yes, Chin is correct. If Alka couldn't make 6 rows with at least 2 squares in each row, then she certainly couldn't make more than 6 rows with at least 2 squares in each row.

4. If Alka had used Chin's rule, she would have tried only up to seven rows.

5.

Number of Rows Attempted	2	3	4
Picture			

Number of Rows Attempted	5	6	7	8	9
Picture					

The number 17 is a prime number. You only have to try making up to 9 rows. You can't make 9 rows with at least 2 squares in each row, so you couldn't make more than 9 rows with at least 2 squares in each row.

6. Mandy knows this because 11 ÷ 6 is less than 2. This means that she cannot make 6 rows with at least 2 squares in each row.

7.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
2	$13 \div 2 = 6$ R1	no	—
3	$13 \div 3 = 4$ R1	no	—
4	$13 \div 4 = 3$ R1	no	—
5	$13 \div 5 = 2$ R3	no	—
6	$13 \div 6 = 2$ R1	no	—
7	$13 \div 7 = 1$ R6	no	—

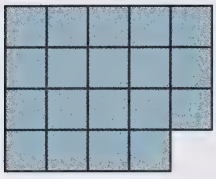
$13 \div 7$ is less than 2. This means that you cannot make 7 or more rows with at least 2 squares in each row.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
2	$17 \div 2 = 8$ R1	no	—
3	$17 \div 3 = 5$ R2	no	—
4	$17 \div 4 = 4$ R1	no	—
5	$17 \div 5 = 3$ R2	no	—
6	$17 \div 6 = 2$ R5	no	—
7	$17 \div 7 = 2$ R3	no	—
8	$17 \div 8 = 2$ R1	no	—
9	$17 \div 9 = 1$ R8	no	—

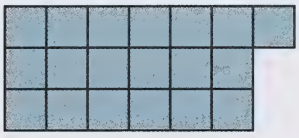
$17 \div 9$ is less than 2. This means that you cannot make 9 or more rows with at least 2 squares in each row.

Activity 3

1. To see if 19 is a prime or composite number, Rhadia could start by making 4 rows of 4 photos plus 3 more photos.



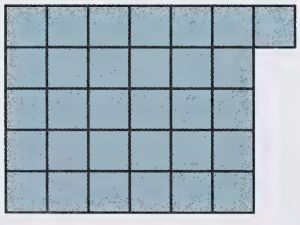
Then she could make 3 rows of 6 photos plus 1 more photo.



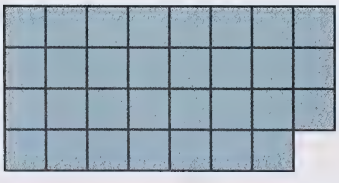
Then she could make 2 rows of 9 photos plus 1 more photo.

Rhadia would be able to see at this point that 19 is a prime number.

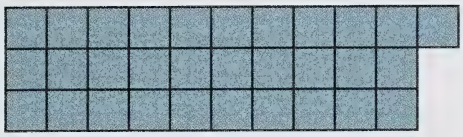
To see if 31 is a prime or composite number, Rhadia could start by making 5 rows of 6 photos plus 1 more photo.



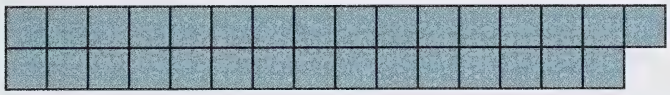
Then, she could make 4 rows of 7 photos plus 3 more photos.



Then, she could make 3 rows of 10 photos plus 1 more photo.



Then, she could make 2 rows of 15 photos plus 1 more photo.



Rhadia would be able to see at this point that 31 is a prime number.

2.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
2	$19 \div 2 = 9$ R1	no	—
3	$19 \div 3 = 6$ R1	no	—
4	$19 \div 4 = 4$ R3	no	—

Mandy has tried enough factors to decide that 19 is a prime number. The closest she can come to forming a square is 4 rows of 4 squares with 3 squares left over.

Number Tried	Test it using division.	Is it a factor?	What is the other factor in the pair?
2	$31 \div 2 = 15$ R1	no	—
3	$31 \div 3 = 10$ R1	no	—
4	$31 \div 4 = 7$ R3	no	—
5	$31 \div 5 = 6$ R1	no	—
6	$31 \div 6 = 5$ R1	no	—

Mandy has tried enough factors to decide that 31 is a prime number. The closest she can come to forming a square is 5 rows of 6 squares with 1 square left over.

Challenge Activity

1.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

The prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

2. The prime pairs and their prime sums are given

$2+3=5$ $2+17=19$

$2+7=7$ $2+29=31$

$2+11=13$ $2+41=43$

3. One of the numbers in every prime pair must be 2. All prime numbers except 2 are odd. For the sum of two numbers to be odd, one of the numbers must be even and the other number must be odd. The only even prime number is 2, so 2 must be one of the numbers in every prime pair.

Keystrokes

1. **a.** $101\times23=$ **2323**

$101\times45=$ **4545**

$101\times67=$ **6767**

b. To find the product of 101 and any 2-digit number, write the two digits and then repeat them.

c. $101\times36=$ **3636**

$101\times87=$ **8787**

$101\times99=$ **9999**

2. a. $37\,037 \times 3 = \mathbf{111\,111}$

$37\,037 \times 6 = \mathbf{222\,222}$

$37\,037 \times 9 = \mathbf{333\,333}$

b. To find the product of 37 037 and a multiple of 3, divide the multiple of 3 by 3 and then write the number you get six times.

$37\,037 \times 12 = \mathbf{444\,444}$

$37\,037 \times 15 = \mathbf{555\,555}$

$37\,037 \times 18 = \mathbf{666\,666}$

3. a. One example was given. Here is another example that shows counting by 19s.

Keystrokes	ON/C	19	+	19	=	=	=	=	=	=	=	=	=
Display	0	19	19	19	38	57	76	95	114	133	152	171	190

Note: You may have to press the + key twice to make the constant work.

b. One example was given. Here's another example.

Method 1

Keystrokes	ON/C	13	×	35	=	−	13	=	=	=
Display	0	13	13	35	455	455	13	442	429	416

Method 2

Keystrokes	ON/C	13	×	35	=	M+	13	−	−	MR	=	=	=
Display	0	13	13	35	455	455	13	13	13	455	442	429	416

Review

1. Textbook, page 38, Skill Bank from This Unit, questions 1.a. to 1.d.

1. a. To find the next number, add 2 to the previous number.
1, 3, 5, 7, **9**, **11**, **13**

b. To find the next number, add 7 to the previous number.
6, 13, 20, 27, **34**, **41**, **48**

c. To find the next number, subtract 3 from the previous number.
33, 30, 27, 24, **21**, **18**, **15**

d. To find the next number, multiply the previous number by 3.
3, 9, 27, 81, **243**, **729**, **2187**

2. Textbook, page 38, Skill Bank from This Unit, questions 2.a. to 2.d.

2. a. Rule: Multiply the input number by 3.

Input	Output
7	21
8	24
9	27

b. Rule: Add 7 to the input number.

Input	Output
12	19
14	21
16	23

c. Rule: Subtract 1 from the input number, and then multiply the difference by 2. Or, double the input number and then subtract 2 from the product.

Input	Output
7	12
8	14
9	16

d. Rule: Multiply the input number by 5 and then subtract 6 from the product.

Input	Output
7	29
8	34
9	39

3. Textbook, page 39, Skill Bank from This Unit, questions 4.a. to 4.d.

4. a. To find the next number, add 1 more to the last number than what you added the time just before.

5, 6, 8, 11, 15, 20, **26**, **33**, **41**
+1 +2 +3 +4 +5 +6 +7 +8

b. To find the next number, subtract 2 more from the last number than what you subtracted the time just before.

100, 98, 94, 88, 80, 70, **58**, **44**, **28**
−2 −4 −6 −8 −10 −12 −14 −16

c. To find the next number, add 2 more to the last number than what you added the time just before.

2, 4, 8, 14, 22, 32, **44**, **58**, **74**
+2 +4 +6 +8 +10 +12 +14 +16

d. To find the next number, subtract 1 more from the last number than what you subtracted the time just before.

50, 49, 47, 44, 40, 35, **29**, **22**, **14**
−1 −2 −3 −4 −5 −6 −7 −8

4. Textbook, page 36, Problem Bank, questions 1.a. and 1.b.

1. a. When you add all 7 numbers in a row (Sunday to Saturday dates) and divide the sum by 7, the answer is always the number in the middle of the row (the Wednesday date).

b. When the numbers at diagonally opposite corners are multiplied together and the smaller product is subtracted from the larger, the difference is always 7.

5. Textbook, page 59, Skill Bank Looking Back, question 1

1. First T-table: Subtract 9 from the input number.

Input	Output
13	4
12	3
11	2

Second T-table: Multiply the input number by 3 and then add 2.

Input	Output
13	41
12	38
11	35

6. a. Each time, the next picture is made by adding 1 more row and 1 more column.

b.

Design Number	Number of Squares in the Design
1	$1 \times 2 = 2$
2	$2 \times 3 = 6$
3	$3 \times 4 = 12$
4	$4 \times 5 = 20$
5	$5 \times 6 = 30$
6	$6 \times 7 = 42$
7	$7 \times 8 = 56$
8	$8 \times 9 = 72$
9	$9 \times 10 = 90$
10	$10 \times 11 = 110$

c. To find the number of squares in each design, multiply the design number by the number one greater than itself.

7. Textbook, page 37, Problem Bank, question 4

Janet folded the paper seven times to divide it into 128 equal parts.

Number of Folds	Number of Equal Parts
1	2
2	4
3	8
4	16
5	32
6	64
7	128

8. a.

b. Each time the pattern grows, draw a new bottom row of dots that has one more dot than the previous bottom row.

The total number of dots is equal to the sum of the design number and the previous total number of dots.

c.

Design Number	Total Number of Dots
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45

d. To find the total number of dots, multiply the design number by the number that is one more than the design number, and then divide the product by 2.

9. a.

Number of Sports Teams	1	2	3	4	5	6	7	8	9	10
Number of Police Officers	6	10	14	18	22	26	30	34	38	42

b. The number of police officers needed is 4 times the number of sports teams plus 2. Another way to say this is that the number of police officers needed is 2 more than quadruple the number of sports teams.

10. You can make change for 50¢ ten different ways.

Quarters	Dimes	Nickels
2	0	0
1	2	1
1	1	3
1	0	5
0	5	0
0	4	2
0	3	4
0	2	6
0	1	8
0	0	10

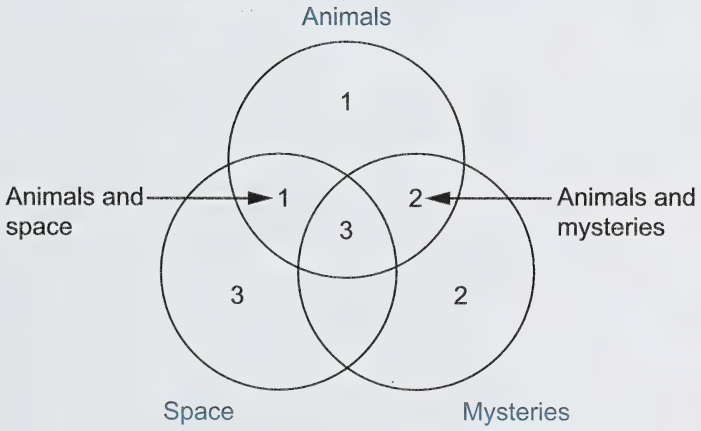
11. The sedan was red, the station wagon was black, and the van was blue. Your strategy should be similar to the following:

- The sedan was not green. Put **no** in the sedan row under green.
- There were no vans available in black. Put **no** in the van row under black.

- The colours of the station wagon and the van begin with the same letter. The two colours that begin with the same letter are blue and black. Since the van is not black, the station wagon must be black and the van must be blue. Put **yes** in the van row under blue and put **yes** in the station wagon row under black. The sedan must be red. Put **yes** in the sedan row under red.

	Red	Blue	Green	Black
Sedan	yes	no	no	no
Station Wagon	no	no	no	yes
Van	no	yes	no	no

12. a.



b. One child chose only an animal book. All the numbers inside the animals circle must have a sum of 7.

c. No children chose only mystery and space books. All the numbers inside the mysteries circle must have a sum of 7.

d. Three children chose only space books. All the numbers inside the space circle must have a sum of 7.

e. Five children didn't choose animal books. Find the sum of all the numbers outside the animal circle.

$3 + 2 = 5$

f. Twelve children in total checked out books. Add the numbers in all 7 areas.

$1+1+2+3+3+2+0=13$

13. a. Textbook, page 37, Problem Bank, question 2

From 7:00 P.M. to midnight is 5 hours at $3.00 per hour.
$5 \times \$3.00 = \$15.00$

From midnight to 1:00 A.M. is 1 hour.
$\$19.00 - \$15.00 = \$4.00$ for 1 hour

From 8:00 P.M. to midnight is 4 hours at $3.00 per hour.
$4 \times \$3.00 = \$12.00$

From midnight to 2:00 A.M. is 2 hours.
$\$20.00 - \$12.00 = \$8.00$ for 2 hours

Andrea charges $4.00 for every hour after midnight.

From 9:00 P.M. to midnight is 3 hours at $3.00 per hour.
$3 \times \$3.00 = \$9.00$

From midnight to 2:00 A.M. is 2 hours at $4.00 per hour.
$2 \times \$4.00 = \$8.00$

$\$9.00 + \$8.00 = \$17.00$

Andrea charged $17.00 from 9:00 P.M. to 2:00 A.M.

b. Textbook, page 37, Problem Bank, question 3

Naveen must walk at least 8 km to receive a contribution greater than $20.

Kilometres Walked	Total Amount Contributed ($)
1	$\$5+(1\times\$2)=\$7$
2	$\$5+(2\times\$2)=\$9$
3	$\$5+(2\times\$3)=\$11$
4	$\$5+(2\times\$4)=\$13$
5	$\$5+(2\times\$5)=\$15$
6	$\$5+(2\times\$6)=\$17$
7	$\$5+(2\times\$7)=\$19$
8	$\$5+(2\times\$8)=\$21$

14. Penny could buy 3 erasers and 1 pencil, 2 erasers and 2 pencils, or 1 eraser and 3 pencils.

Erasers (20¢)	Pencils (15¢)	Total Amount Spent
3	1	$(3\times 20¢)+(1\times 15¢)=60¢+15¢=75¢$
2	2	$(2\times 20¢)+(2\times 15¢)=40¢+30¢=70¢$
1	3	$(1\times 20¢)+(3\times 15¢)=20¢+45¢=65¢$

15. a.

	Jenny	Joan	Jill
$4	no		yes
$7	yes	no	
$10		yes	

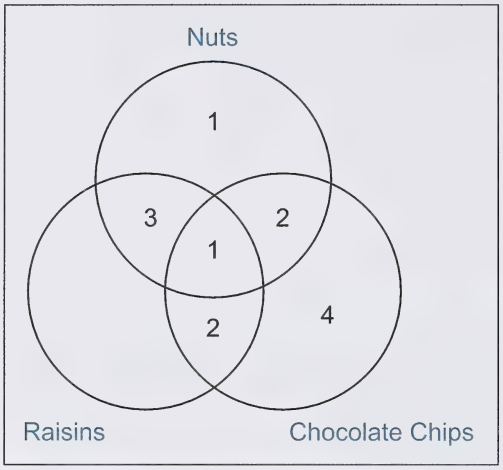
- Jenny saved more than Jill. Jenny could not have saved $4. Put **no** in the $4 row under her name.
- Joan saved an even number of dollars. Joan could not have saved $7. Put **no** in the $7 row under her name.

- Jenny saved less than Joan. Jenny must have saved $7. Put **yes** in the $7 row under her name.

- Jenny saved more than Jill, so Jill must have saved $4. Put **yes** in the $4 row under her name. Joan must have saved $10. Put **yes** in the $10 row under her name

b. Jenny saved $7, Joan saved $10, and Jill saved $4.

16. a.



b. One bar had only nuts. All the areas inside the nuts circle must have a sum of 7.

c. Two bars had only raisins and chocolate chips. All the areas inside the chocolate chip circle must have a sum of 9.

d. No bars had only raisins. All the areas inside the raisins circle must have a sum of 6.

e. Seven bars didn't have raisins. Add all the numbers outside the raisins circle.

$1+2+4=7$

f. There is a total of 13 granola bars in the bag. Add the numbers in all the areas.

$1+2+1+3+0+2+4=13$

17. Textbook, page 88, Problem Bank, question 3

A 3	B 6		C 4	D 8
	E 3	F 5		1
G 6		H 4	I 2	
J 4	K 2		L 1	M 8*
	N 2	0		0*

* **Note:** M could also be 24.

- If A is a multiple of 6, it could be 30 or 36; but B must be a multiple of 7, so A is 36 and B is 63.
- If C is a multiple of 8, it could be 40 or 48; but D must be a multiple of 9, so C is 48 and D is 81.
- If E is a multiple of 5, it could be 30 or 35; but F must be a multiple of 6, so A is 35 and B is 54.
- If G is a multiple of 8, it must be 64 and J must be 42.
- If K is a multiple of 11, it must be 22 and N must be 20, so A is 35 and B is 54.
- If H is a multiple of 3, it could be 42, 45 or 48; but I must be a multiple of 7, so H is 42 and I is 21.
- If L is a multiple of 6, it could be 12, or 18; but M must be a multiple of 8, so there are two possible answers: If L is 12, then M is 24; If L is 18, then M is 80.

18. To find the first ten multiples of 6, begin with 6 and keep adding 6 to get the next multiple. The first ten multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, and 60.

19. No, 66 is not a multiple of 4 because $66 \div 4 = 16 \text{ R}2$. If 4 was a factor of 66, it would divide evenly into 66 without a remainder.

20. **a.** The numbers are all multiples of 7.
b. If the pattern continues, the next three numbers are 49, 56, and 63.

21. Textbook, page 88, Problem Bank, questions 1 and 2

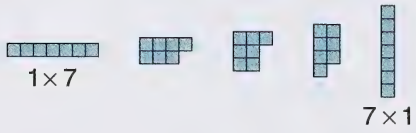
1.

	Composite Number	Prime Numbers
Even Numbers	4, 6, 8, 10, 12, 14, 16, 18, 20	2
Odd Numbers	9, 15	3, 5, 7, 11, 13, 17, 19

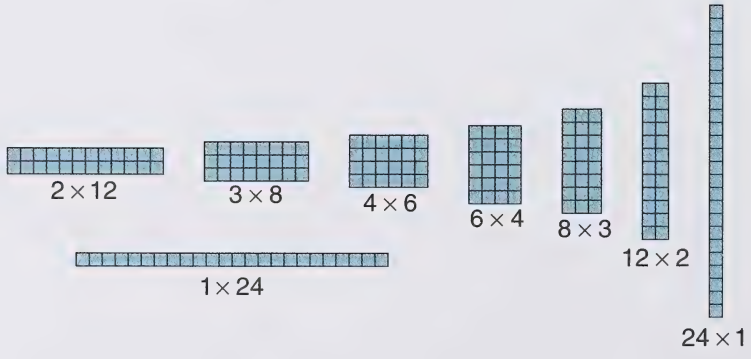
	Prime Numbers	Composite Numbers
Factors of 12	2, 3	4, 6, 12
Not Factors of 12	5, 7, 11, 13, 17, 19	8, 9, 10, 14, 15, 16, 18, 20

Note that 1 is neither a prime number nor a composite number.

2. The number 7 is not a composite number because you can only draw one rectangular array for 7.



The number 24 is not a prime number because you can make arrays of 24 squares that form more than a single line.



22. Textbook, page 67, Practise Your Skills, questions 1, 2, and 3

1. The factors of 70 are 1, 2, 5, 7, 10, 14, 35, and 70.
   The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
   The factors of 96 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, and 96.
   The factors of 55 are 1, 5, 11, and 55.
   The factors of 28 are 1, 2, 4, 7, 14, and 28.
   The factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, and 100.
   The factors of 67 are 1 and 67.
   The factors of 76 are 1, 2, 4, 19, 38, and 76.
   The factors of 35 are 1, 5, 7, and 35.

2. The only number that is prime is 67.

3. The numbers that are multiples of 7 are 70, 28, and 35.

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Cut-Out Learning Aids

Hundred Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100